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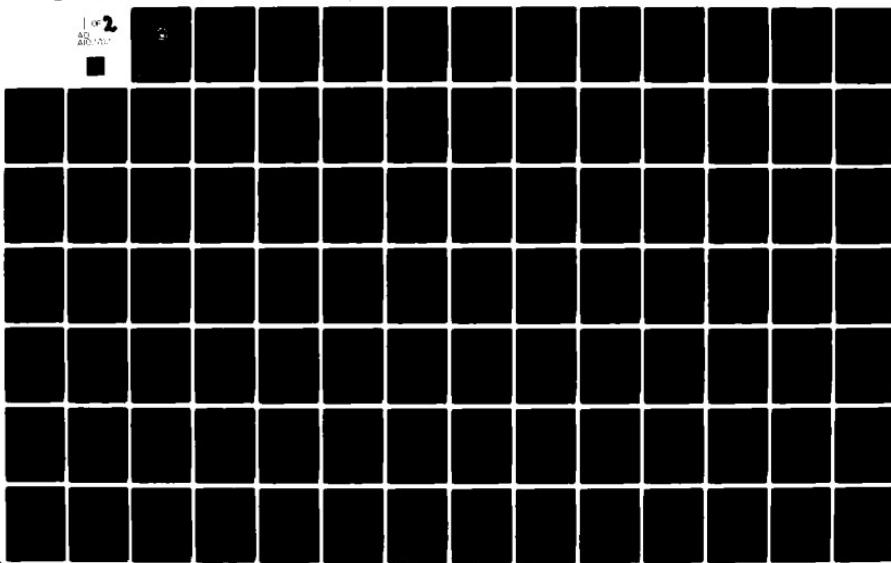
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THESIS

AN ADAPTIVE KALMAN IDENTIFIER
AND ITS APPLICATION TO
LINEAR AND NON-LINEAR ARMA MODELING.

by

Leopoldo M. Mayoral

11 Mar 1981

Thesis Advisor:

S.R. Parker

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#20 - ABSTRACT - (CONTINUED)

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It is shown that the Adaptive least mean square (LMS) Adaptive Recursive LMS and Adaptive Lattice filters are special suboptimal cases of the AKI. The convergence and modeling properties are compared with those of the AKI by simulation using various types of data.

With minor modification, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.

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An Adaptive Kalman Identifier
and Its Application to
Linear and Non-Linear ARMA Modeling

by

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ABSTRACT

The problem of accurately replicating the parameters which define a given system for the purposes of implementing modern control strategies is important. Using an Autoregressive-Moving Average (ARMA) representation for the unknown system, a model is identified by processing input/output data to estimate the coefficients associated with the ARMA equation. Identification of unknown system parameters using Kalman filtering methods was accomplished by augmenting the state vector. In this thesis the Kalman filter is formulated so that parameters can be identified explicitly. We call this approach the Adaptive Kalman Identifier (AKI).

It is shown that the Adaptive least mean square (LMS), Adaptive Recursive LMS and Adaptive Lattice filters are special suboptimal cases of the AKI. The convergence and modeling properties are compared with those of the AKI by simulation using various types of data.

With minor modifications, the AKI algorithm was used to identify the linear and non-linear ARMA models of the phase locked loop (PLL). A discrete PLL using a forward Euler integration scheme was used as a source of non-linear data. The AKI technique appears to enable one to discern when a potential non-linear system enters into its non-linear mode of operation.

TABLE OF CONTENTS

I.	INTRODUCTION -----	13
	A. BACKGROUND -----	13
	B. INVESTIGATIONS AND CONTRIBUTIONS -----	16
	C. ORGANIZATION -----	17
II.	FORMULATION OF THE PROBLEM -----	18
	A. DETERMINISTIC CASE -----	18
	B. NON DETERMINISTIC CASE -----	20
	1. Autoregressive Form ($a_i = 0$) -----	24
	2. Moving Average Form ($b_i = 0$) -----	27
	3. Autoregressive-Moving Average Form -----	30
	C. OBSERVATIONS -----	33
III.	COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND WIDROW LMS ADAPTIVE FILTER -----	36
	A. PRELIMINARIES -----	36
	B. THE COMPARISON -----	38
	C. OBSERVATIONS -----	41
IV.	COMPARISON BETWEEN THE ADAPTIVE KALMAN IDENTIFIER AND FEINTUCH ADAPTIVE RECURSIVE LMS FILTER -----	43
	A. PRELIMINARIES -----	43
	B. THE COMPARISON -----	47
	C. OBSERVATIONS -----	53
V.	SYNTHETIC DATA GENERATING PLANTS -----	54
	A. AUTOREGRESSIVE-MOVING AVERAGE DATA -----	54
	1. Moving Average Data (Case 1) -----	55
	2. Autoregressive Data (Case 2) -----	55
	3. Autoregressive-Moving Average Data (Case 3) -----	56

B.	PHASE LOCKED LOOP DATA -----	57
VI.	SOFTWARE -----	67
A.	ADAPTIVE KALMAN IDENTIFIER -----	67
B.	LMS ADAPTIVE FILTER -----	69
C.	ADAPTIVE RECURSIVE LMS FILTER -----	73
VII.	NON-LINEAR IDENTIFICATION -----	75
VIII.	FINDINGS, CONCLUSION -----	81
	A. ORDER OF THE UNKNOWN SYSTEM IS KNOWN -----	81
	1. AKI vs LMS Adaptive Filter -----	81
	2. AKI Applied to Autoregressive Data -----	91
	3. AKI Applied to ARMA Data -----	93
	4. AKI vs Adaptive Recursive LMS Filter -----	98
	5. AKI Applied to PLL Data (Linear Region) -	103
	6. AKI Applied to PLL Data (Non-linear Region) -----	103
	B. ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING) -----	109
	1. AKI vs LMS Adaptive Filter -----	109
	2. AKI Applied to AR and ARMA Data -----	114
	C. CONCLUSION -----	115
	D. TOPICS FOR FURTHER CONSIDERATION -----	118
APPENDIX A:	The Discrete Weiner Problem -----	120
APPENDIX B:	Program ARMA -----	124
APPENDIX C:	Program PLL -----	126
APPENDIX D:	Program ADAPTSN -----	131
APPENDIX E:	Program LMS -----	147
APPENDIX F:	Program LMSR -----	151

LIST OF REFERENCES -----	158
INITIAL DISTRIBUTION LIST -----	161

LIST OF TABLES

TABLE

6.1	AKI Flag Meanings -----	68
6.2	Valid Flag Combinations -----	69
7.1	Non-linear Weighting Coefficients Calculated Using Three Methods -----	80
8.1	Actual vs AKI Estimates for Coefficients of Equation 8.3 -----	93
8.2	Actual vs AKI Estimates for Coefficients of Equation 8.4 at the 371st Iteration -----	94
8.3	Actual vs AKI and Adaptive Recursive LMS Filter Estimates for Coefficients of Equation 8.6 -----	100
8.4	Actual vs AKI Estimates of the ARMA Repre- sentation for the PLL (Linear Region) -----	103
8.5	Overmodeling of a MA Process Using the AKI ($k = 250$) -----	113
8.6	Overmodeling of a MA Process Using the AKI of orders AR = 2, MA = 6 -----	113

LIST OF FIGURES

FIGURE

2.1	Adaptive Kalman Identifier -----	34
3.1	LMS Adaptive Filter Implementation of System Identification -----	37
4.1	Adaptive Recursive LMS Filter Applied to System Identification -----	48
5.1	ARMA Digital Plant -----	56
5.2	PLL Block Diagram -----	58
5.3	Final PLL Block Diagram -----	59
5.4	PLL Step Response, Linear Region -----	60
5.5	PLL Step Response, Non-linear Region -----	61
5.6	Discrete PLL Block Diagram -----	62
5.7	Programmable PLL Implementing Forward Euler Integration -----	63
5.8	Discrete PLL Step Response, Linear Region -----	64
5.9	Discrete PLL Step Response, Non-linear Region -----	65
6.1	AKI General Program Flow -----	70
6.2	LMS Filter Program Flow -----	72
6.3	LMSR Filter Program Flow -----	74
7.1	PLL, Third Order Taylor Series Approximation -	76
8.1	LMS Adaptive Filter Weights, $k_s = -.200$ -----	83
8.2	LMS Adaptive Filter Weights, $k_s = -.300$ -----	84
8.3	LMS Adaptive Filter Weights, $k_s = -.400$ -----	85
8.4	LMS Adaptive Filter Weights, $k_s = -.500$ -----	86
8.5a	LMS Adaptive Filter Weights, $k_s = -.600$ -----	87

FIGURE

8.5b	LMS Adaptive Filter Weights, $k = -.200$, with Increased Measurement Noise -----	88
8.6	AKI Moving Average Plant Coefficients -----	89
8.7	LMS Adaptive Filter Weights -----	89
8.8	AKI Averaged Gains for Moving Average Model --	90
8.9	AKI Moving Average Instantaneous Gains -----	90
8.10	AKI Autoregressive Plant Coefficients -----	92
8.11	AKI Autoregressive Averaged Gains -----	92
8.12	AKI(4,5) Computed Coefficients, Perry's Example -----	95
8.13	AKI(4,5) Average Gain Values -----	95
8.14a	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) ----	96
8.14b	Pole/zero Models Produced by AKI and Lattice Algorithms for a Plant with the Character- istic Transfer Function of Equation (8.4) at the 371st Iteration -----	97
8.15	Instantaneous Gain Values, AKI(4,5) Model ----	98
8.16	AKI(2,2) Computed Coefficients, Feintuch Example -----	101
8.17	LMSR Computed Coefficients, Feintuch Example -	101
8.18	AKI(2,2) Averaged Gains, Feintuch Example ----	102
8.19	Pole/zero Models Produced by AKI and Adaptive Recursive LMS Algorithms for a Plant with the Characteristic Transfer Function of Equation 8.6 -----	102
8.20	PLL, AKI(2,3) Computed Coefficients -----	104
8.21	Averaged Gain Values, AKI(2,3) PLL Model ----	104
8.22	PLL Root Locus Analysis vs Computed Roots of the AKI ARMA Model -----	106

FIGURE

8.23	Variation of \bar{a} vs Input Noise Signal Power ---	108
8.24	Comparison of $\frac{\sin x}{x}$ and P.D.F. of Input Noise Signal -----	110
8.25a	AKI(0,4) Model of a Third Order MA Process ----	112
8.25b	Fourth Order LMS Filter Weights of a Third Order MA Process -----	112
8.26	AKI(3,1) Overmodeled AKI(4,2) -----	116
8.27	AKI(3,1) Overmodeled AKI(5,5) -----	116
8.28	AKI(4,5) Overmodeled AKI(5,6) -----	117
A.1	The Discrete Weiner Problem -----	121

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I. INTRODUCTION

The accomplishments in the area of microprocessor technology in the last decade have made a noticeable impact in the area of modern control. The desire to implement modern control theories taking advantage of these advancements, has made it imperative that the engineer attempt to mathematically replicate the system he ultimately desires to control. Efforts in this regard have, hence, generated a growing interest in the area of system identification [Ref. 1,2,3,4]. By implication this thesis concerns itself with discrete/digital signal processing.

A. BACKGROUND

System identification or modeling can be accomplished by innovative application of existing techniques which were generally considered filtering or state estimation methods [Ref. 5]. Previous research efforts have, for obvious reasons, focused on linear modeling; however, there is a rising interest in non-linear modeling methods [Ref. 6,7,8].

An attractive form for the representation of an unknown system is the Autoregressive-Moving Average (ARMA) equation,

$$y(k) = \sum_{i=0}^{\infty} a_i u(k-i) - \sum_{i=1}^{\infty} b_i y(k-i) \quad (1.1)$$

which states that the present output, $y(k)$, is a linear combination of past outputs, $y(k-i)$, and of past and present

inputs, $u(k)$. Its attractiveness lies in its linear character and easily implementable structure using microprocessor/computer algorithms. Its relationship to the state space representation of a linear system has already been established [Ref. 9,10] making it germane to consider that a system is identifiable by an equation of the form (1.1). The system identification problem thus entails identifying the coefficients a_i and b_i .

There are several methods for computing the coefficients, a_i and b_i , however, it is not the intent of this brief introduction to attempt to develop even the majority of them. Nevertheless, it is practical to present a referenced history of those methods encountered in this thesis.

Adaptive algorithms for the purpose of estimating the coefficients of (1.1) have always been of interest. Widrow, using a least means square error criterion and implementing the method of steepest descent, developed an adaptive algorithm which estimated the coefficients of the moving average process associated with equation (1.1) [Ref. 11]. That is, the moving average model,

$$y(k) = a'_0 u(k) + a'_1 u(k-1) + a'_2 u(k-2) + \dots \quad (1.2)$$

where,

$$a'_0 = a_0$$

$$a'_1 = (a_1 - a_0 b_1)$$

$$a'_2 = (a_2 - a_0 b_2) - b_1 (a_1 - a_0 b_1)$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

was identified. The LMS theory has since been extended to include block LMS filtering methods [Ref. 12,13,14] using various search techniques [Ref. 15,4(Chapter 5)]. These methods have enjoyed much popularity in the area of Linear Prediction and digital speech processing [Ref. 16,17].

The shortcoming of representing equation (1.1) by its equivalent moving average model (1.2) lies in the practical aspect of its implementation. That is, the infinite series represented by equation (1.2) must be truncated at some point resulting in an approximation which may not adequately represent equation (1.1). Hence, efforts to adaptively estimate the coefficients for both the autoregressive (b_i) and moving average (a_i) processes continued [Ref. 18,19,20] with various degrees of success. Feintuch's Adaptive Regressive LMS procedure [Ref. 18] is still a controversial issue [Ref. 19,20].

From yet a different direction, Anderson and Moore suggested that the Kalman filtering algorithms can be used as a means to identify the a_i and b_i coefficients¹ of equation (1.1) [Ref. 21, pp. 50-52]. This thesis exploits this application.

¹Anderson and Moore in fact formulate the technique to compute the coefficients a_i, b_i $i = 1, 2, \dots, p$ where $p = n+m$.

B. INVESTIGATIONS AND CONTRIBUTIONS

This thesis formulates an adaptive application of the Kalman filter to identify the coefficients of the general ARMA equation (1.1). It is shown that the LMS adaptive filter [Ref. 11] and the Adaptive Recursive LMS filter [Ref. 18] are (1) special cases of the Adaptive ARMA Kalman identifier and (2) sub-optimal with respect to the underlying least means square (LMS) error criterion upon which the LMS adaptive filter, the adaptive recursive filter and the Kalman filter are based. It is also demonstrated that the adaptive Kalman identifier has excellent convergence properties.

By comparison, it is noted that the Kalman algorithm accounts for measurement noise where the LMS algorithms do not, a heretofore unapproached problem by LMS algorithms. The results indicate that the suggested modification by Griffiths [Ref. 22] of the convergence factor, k_s , for the LMS adaptive filter is justified.

The application of the Kalman filter algorithm is extended to identification of the coefficients associated with a special case of the generalized non-linear ARMA model [Ref. 8]. The results of the non-linear simulations suggest a technique for determining when a potential non-linear system enters its non-linear operating regime. Such a technique can prove valuable when on-line performance evaluation of a known non-linear system is required.

Lastly, the connection between the Kalman filter algorithm and the lattice filter algorithm [Ref. 6] is made. An example

which demonstrates and compares the performance of both algorithms is given.

C. ORGANIZATION

Chapter II presents and exploits the Kalman filter equations emphasizing its connections with the Yule-Walker and the discrete Weiner-Hopf equations. The theory is further developed to investigate the general ARMA case. Chapter III pursues the theoretical comparisons between the discrete Weiner-Hopf equation upon which the adaptive LMS filter is based and the MA form of the Kalman filter. The theoretical comparison between the Adaptive Recursive LMS filter and the general ARMA form of the Kalman filter is made in Chapter IV. The software methods by which linear and non-linear synthetic data is generated are discussed in Chapter V. A short user's description of the data processing programs and the options provided is given in Chapter VI. A non-linear application for the identification of the parameters of a non-linear plant is presented in Chapter VII followed by a discussion and presentation of the results in Chapter VIII.

II. FORMULATION OF THE PROBLEM

Identifying the coefficients of the Autoregressive-Moving Average (ARMA) equation,

$$y(k) + b_1 y(k-1) + \dots + b_n y(k-n) = a_0 u(k) + a_1 u(k-1) + \dots + a_m u(k-m) \quad (2.1a)$$

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) \quad (2.1b)$$

can be formulated as an adaptive Kalman identification problem. That is, instead of using the well known Kalman Filter equations [Ref. 23,24] to recursively estimate the states of a system, one can utilize the Kalman filter equations to adaptively estimate the coefficients of either an Autoregressive (AR), Moving Average (MA), or Autoregressive-Moving Average (ARMA) process by proper definition of the quantities involved. We call this the Adaptive Kalman Identifier for obvious reasons.

A. THE DETERMINISTIC CASE

Consider for the moment that the a_i and b_i are constant. Then by collecting a sufficient number of measurements of the input $u(k)$ and the output $y(k)$, one can readily solve a set of linear equations for the a_i and b_i coefficients. The "sufficient number" that is needed is $n+m+1$ which define the

$n+m+1$ linear equations which solve the $n+m+1$ unknown coefficients. The matrix equation to be solved takes the form:

$$\begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(m) \\ y(m+1) \\ \vdots \\ y(m+n) \end{bmatrix} = \begin{bmatrix} u(0) & 0 & \dots & 0 & 0 & \dots & \dots & \dots & 0 \\ u(1) & u(0) & & & y(0) & & & & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & & & & \vdots \\ u(m) & u(m-1) & u(0) & y(m-1) & y(m-2) & y(0) & 0 & a_0 \\ \hline u(m+1) & & u(1) & y(m) & y(m-1) & y(1) & 0 & a_1 \\ & & & & & & & \vdots \\ & & & & & & & a_m \\ u(m+n) & \dots & u(m) & y(n-1) & y(n-2) & y(0) & \vdots & b_1 \\ & & & & & & & b_2 \\ & & & & & & & \vdots \\ & & & & & & & b_n \end{bmatrix} \quad (2.2)$$

$$Y = T \begin{bmatrix} a \\ \hline \cdots \\ b \end{bmatrix} \quad (2.3)$$

It is interesting to note that the $(n+m+1) \times (n+m+1)$ matrix, T , is block symmetric. The solution to (2.3) is readily apparent, namely,

$$\begin{bmatrix} a \\ \hline \cdots \\ b \end{bmatrix} = T^{-1} Y \quad (2.4)$$

where T^{-1} is the inverse matrix of T . Since we have assumed that the elements of T are perfect noiseless measurements, and that we somehow know a-priori the number of unknown

coefficients, then T is of full rank, namely $\text{rank}(T) = n+m+1$, and the inverse of T exists.

B. THE NONDETERMINISTIC CASE

Rarely can the coefficients be modeled so ideally. A more prudent and realistic model admits that the ARMA equation coefficients are subject to random perturbations. Further, it can be said that in general measurement devices introduce noise into the measurement data. Hence, the measurement model should reflect this fact. Developing the Adaptive Kalman Identifier along the same lines as [Ref. 21] we let,

$$a_i(k+1) = a_i(k) + w_i(k) \quad i = 0, 1, 2, \dots, m \quad (2.5a)$$

$$b_j(k+1) = b_j(k) + w_j(k) \quad j = 1, 2, \dots, n \quad (2.5b)$$

where the $\{w_i(k), w_j(k)\}$ are samples from a zero mean, white, gaussian random process. Additionally, we assume that the noise sources are uncorrelated,

$$E[w_{a_r}(k)w_{a_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6a)$$

$$E[w_{b_r}(k)w_{b_s}(k)] = 0 \quad \text{for } r \neq s \quad (2.6b)$$

$$E[w_{a_r}(k)w_{b_s}(k)] = 0 \quad \text{for all } r, s \quad (2.6c)$$

Similarly, allowing for noisy measurement devices, the measurement equation (2.1b), is modelled as,

$$y(k) = \sum_{i=0}^m a_i u(k-i) - \sum_{i=1}^n b_i y(k-i) + v(k) \quad (2.7)$$

where the $\{v(k)\}$ are samples from a zero mean, white, gaussian random process. It is also assumed that,

$$E[w_i(k) v(k)] = 0 \text{ for all } i \quad (2.8a)$$

$$E[w_j(k) v(k)] = 0 \text{ for all } j \quad (2.8b)$$

That is, it is assumed that the noise perturbations associated with one coefficient are independent of the noise perturbations associated with any other coefficient, and that the measurement noise and the coefficient perturbations are independent.

At this point it is necessary to use judgment and experience and utilize all the information known about the physical system which equation (2.7) represents to assign variances for the random processes $\{w_i(k)\}$, $\{w_j(k)\}$ and $\{v(k)\}$. Let

$$Q = E \left\{ \begin{bmatrix} w_i(k) \\ w_j(k) \end{bmatrix} [w_i(k) \ w_j(k)] \right\} \quad (2.9a)$$

$$Q = \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \quad (2.9b)$$

where, $Q_1 = \text{diag } (\sigma_{w_i}^2)$, $Q_2 = \text{diag } (\sigma_{w_j}^2)$ and,

$$R = E[v(k) v(k)] \quad (2.9c)$$

and $E[x]$ is the expected or average value of x .

Instead of using x 's denoting the commonly used notation for state variables, we retain the flavor of the problem by using the a_i 's and b_i 's as the "states" of the Adaptive Kalman Identifier. It is hoped that a more meaningful understanding of the Adaptive Kalman Identifier may thus be gained. Therefore, define the state vector,

$$\begin{bmatrix} a_0(k) \\ a_1(k) \\ \vdots \\ \vdots \\ a_m(k) \\ \hline b_1(k) \\ b_2(k) \\ \vdots \\ \vdots \\ b_n(k) \end{bmatrix} = \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} \quad (2.10)$$

Combining equations (2.5) and (2.10), we have the discrete, first order Gauss-Markov process [Ref. 3],

$$\begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} = \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} + \underline{w}(k) \quad (2.11)$$

where,

$$\underline{w}(k) = \begin{bmatrix} w_i(k) \\ \hline \vdots \\ \vdots \\ w_j(k) \end{bmatrix} . \quad (2.12)$$

Note that $\underline{w}(k)$ is an $n+m+1$ vector whose elements are a concatenation of the noise sequences associated with the a_i and b_i . To complete the Adaptive Kalman Identifier formulation we define the measurement vector,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m) \ -y(k-1) \ \dots \ -y(k-n)] \quad (2.13)$$

and its associated measurement equation,

$$y(k) = H(k) \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} + v(k). \quad (2.14)$$

Then the solution to the Adaptive Kalman Identifier problem [Ref. 21] is,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = [I - K(k)H(k)] \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k)y(k) \quad (2.15a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (2.15b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.15c)$$

Equations (2.15) are initialized by assuming an initial value for the coefficients (2.10) and assigning to our assumption a measure of our confidence in the initial guess. That is, we pick the values,

$$\begin{bmatrix} \hat{\underline{a}}(0|-1) \\ \hline \hat{\underline{b}}(0|-1) \end{bmatrix} \text{ and } P(0|-1) \quad (2.16)$$

where $P(0|-1)$ is defined as the error covariance of the coefficients,

$$P(k|k-1) = E \left\{ \begin{bmatrix} \underline{a}(k) \\ \vdots \\ \underline{b}(k) \end{bmatrix} - \begin{bmatrix} \hat{\underline{a}}(k|k) \\ \vdots \\ \hat{\underline{b}}(k|k) \end{bmatrix} \begin{bmatrix} \underline{a}(k) \\ \vdots \\ \underline{b}(k) \end{bmatrix}^T - \begin{bmatrix} \hat{\underline{a}}(k|k) \\ \vdots \\ \hat{\underline{b}}(k|k) \end{bmatrix} \right\}^T \quad (2.17)$$

for $k = 0$. Equation (2.15b) is generally referred to as the Kalman gain. Two special cases are of interest: case (1); $a_i = 0$ for all i , and case (2); $b_i = 0$ for all i .

1. Autoregressive Form ($a_i = 0$)

For case 1, equation (2.7) takes the form,

$$y(k) = - \sum_{i=1}^n b_i y(k-i) + v(k). \quad (2.18)$$

This is a recursive equation stating that the present output is a linear combination of past outputs corrupted with additive gaussian noise, $v(k)$. Equation (2.18) is more formally recognized as an autoregressive process of order n [Ref. 25, 26]. Writing the Kalman solution for the coefficients of the AR process,

$$\hat{\underline{b}}(k+1|k) = [I - K(k)K(k)] \hat{\underline{b}}(k|k-1) + K(k)y(k) \quad (2.19a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.19b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.19c)$$

Alternate forms of the Kalman equations given by Maybeck [Ref. 27] are,

$$\begin{aligned} \hat{\underline{b}}(k+1|k) &= P(k|k-1) P(k|k-1)^{-1} \hat{\underline{b}}(k|k-1) \\ &\quad + P(k+1|k) H^T(k) R^{-1} y(k) \end{aligned} \quad (2.20a)$$

$$P(k+1|k) = (P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)^{-1}. \quad (2.20b)$$

We can model the fact that we have no a-priori knowledge about the initial values of the coefficients by letting,

$$[P(0|-1)]^{-1} \approx 0. \quad (2.21)$$

Further, if we remain ignorant and totally doubt our previous estimate, then $P(k|k-1)$ is modeled as,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.22)$$

Equations (2.20) reduce to,

$$\hat{\underline{b}}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} Y(k), \quad (2.23)$$

the weighted least squares estimate, previously encountered by Swerling [Ref. 28,29,30], of the coefficients, \underline{b} . Carrying the analysis further and letting $R^{-1} = I$, we have the Penrose pseudo-inverse solution [Ref. 31,32],

$$\hat{\underline{b}}(k+1|k) = [H(k) H^T(k)]^{-1} H(k) Y(k) \quad (2.24)$$

Swerling [Ref. 28] has shown that if the weighting matrix in (2.23) is not the inverse of the covariance matrix of the measurement errors, then the accuracy of the estimated coefficients (2.24) will be degraded.

The aforementioned notwithstanding, we press further into the analysis of equation (2.24). The product $H(k)H^T(k)$ can be written as,

$$H(k)H^T(k) = \begin{bmatrix} y(k-1) \\ y(k-2) \\ \vdots \\ y(k-n) \end{bmatrix} [y(k-1) \ y(k-2) \ \dots \ y(k-n)] \quad (2.25)$$

$$H(k)H^T(k) = \begin{bmatrix} y^2(k-1) & y(k-1)y(k-2) & \dots & y(k-1)y(k-n) \\ y(k-2)y(k-1) & y^2(k-2) & & \vdots \\ \vdots & & & \vdots \\ y(k-n)y(k-1) & \dots & \dots & y(k-n)y(k-n) \end{bmatrix}. \quad (2.26)$$

As we let $k \rightarrow \infty$ the expected value of (2.26) becomes the autocorrelation matrix,

$$E\{\lim_{k \rightarrow \infty} H(k) H^T(k)\} = R_{yy}(k) \quad (2.27)$$

where,

$$R_{YY}^{(k)} = \begin{bmatrix} R_{YY}^{(0)} & R_{YY}^{(1)} & \dots & R_{YY}^{(n)} \\ R_{YY}^{(-1)} & R_{YY}^{(0)} & \dots & \vdots \\ \vdots & & & \vdots \\ R_{YY}^{(-n)} & \dots & \dots & R_{YY}^{(0)} \end{bmatrix} . \quad (2.28)$$

Similarly, the product $H(k)y(k)$ as $k \rightarrow \infty$ becomes

$$r_{yy}^{(k)} = \begin{bmatrix} R_{YY}^{(-1)} \\ R_{YY}^{(-2)} \\ \vdots \\ R_{YY}^{(-n)} \end{bmatrix} . \quad (2.29)$$

The steady state solution for the estimate of the coefficients $\hat{b}_{ss}(k+1|k)$ is,

$$\hat{b}_{ss}(k+1|k) = R_{YY}^{-1}(k)r_{yy}^{(k)} . \quad (2.30)$$

Equation (2.30) is one of the starting points from which Perry [Ref. 6] develops his Lattice modeling algorithms. Equation (2.30) can also be recognized as the solution to the Yule-Walker or Normal equations [Ref. 26].

2. Moving Average Form ($b_i = 0$)

Referring once again to equation (2.7) and setting the coefficients $b_i = 0$ for all i we have case (2),

$$y(k) = \sum_{i=0}^m u(k-i) + v(k) \quad (2.31)$$

Since $v(k)$ is by definition noise associated with the measurement, we can combine $y(k)$ and $v(k)$ such that,

$$z(k) = y(k) - v(k) = \sum_{i=0}^m a_i u(k-i). \quad (2.32)$$

Equation (2.32) simply states that the present measurement is a linear combination of past and present inputs or by definition, a Moving Average (MA) process [Ref. 25,33]. The Adaptive Kalman Identifier estimate for the coefficients of the MA process is as before,

$$\hat{a}(k+1|k) = [I - K(k)H(k)] \hat{a}(k+1|k) + K(k)z(k) \quad (2.33a)$$

$$K(k) = P(k|k-1) H^T(k) [H(k)P(k|k-1) H^T(k) + R]^{-1} \quad (2.33b)$$

$$P(k+1|k) = P(k|k-1) - K(k)H(k)P(k|k-1) + Q. \quad (2.33c)$$

Using the alternate forms of the above equations we arrive at,

$$\begin{aligned} \hat{a}(k+1|k) &= [P(k+1|k) [P(k|k-1)]^{-1}] \hat{a}(k|k-1) \\ &\quad + [P(k+1|k) H^T(k) R^{-1}] z(k) \end{aligned} \quad (2.34a)$$

$$P(k+1|k) = [(P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)]^{-1}. \quad (2.34b)$$

Arguing as we did for the autoregressive case, no a-priori knowledge about the initial values of the moving average coefficients, implies that,

$$[P(0|-1)]^{-1} \approx 0. \quad (2.35)$$

And even if after we processed one measurement we still admitted no knowledge as to the accuracy of the previous estimate, we imply that,

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.36)$$

Substituting these implications into equations (2.34) our estimate becomes,

$$\hat{a}(k+1|k) = [H(k) R^{-1} H^T(k)]^{-1} H(k) R^{-1} z(k) \quad (2.37)$$

the by now familiar weighted least squares estimate [Ref. 28, 29, 30]. If once again we allow R^{-1} to be unity, let k tend toward infinity and take the expectation of equation (2.37), one arrives at the discrete form of the Wiener-Hopf equation [Ref. 11], namely

$$\hat{a}(k+1|k) = R_{uu}^{-1}(k) r_{uz}(k) \quad (2.38)$$

where it can easily be shown that,

$$R_{uu}(k) = \begin{bmatrix} R_{uu}(0) & R_{uu}(-1) & \dots & R_{uu}(-m) \\ R_{uu}(-1) & R_{uu}(0) & \dots & R_{uu}(1-m) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ R_{uu}(-m) & \dots & & R_{uu}(0) \end{bmatrix} \quad (2.39)$$

and,

$$R_{uy}(k) = \begin{bmatrix} R_{uy}(0) \\ R_{uy}(-1) \\ \vdots \\ \vdots \\ R_{uy}(-m) \end{bmatrix} . \quad (2.40)$$

Equation (2.38) provided another point of departure from which Perry [Ref. 6] develops the MA Lattice modeling algorithms. Appendix A develops the Wiener-Hopf equation otherwise known as the all-zero model from yet another approach.

The all-zero model is fundamental to Widrow least mean square (LMS) adaptive filters [Ref. 11] and linear prediction theory [Ref. 16].

3. Autoregressive-Moving Average Form

Returning to the alternate form (equations 2.20) of the Kalman Filter equations (2.19) the development of the Autoregressive Moving Average (ARMA) Adaptive Kalman Identifier follows. The estimate for the a_i and b_i coefficients is,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = [P(k+1|k) [P(k|k-1)]^{-1}] \begin{bmatrix} \hat{a}(k|k-1) \\ \hat{b}(k|k-1) \end{bmatrix} + [P(k+1|k) H^T(k) R^{-1}] y(k) \quad (2.41a)$$

$$P(k+1|k) = [(P(k|k-1))^{-1} + H^T(k) R^{-1} H(k)]^{-1} . \quad (2.41b)$$

Progressing in the same manner as in the previous two cases, we assume,

$$[P(0|-1)]^{-1} \approx 0 \quad (2.42a)$$

$$[P(k|k-1)]^{-1} \approx 0. \quad (2.42b)$$

The estimate then becomes,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = [H^T(k) R^{-1} H(k)]^{-1} H(k) R^{-1} y(k). \quad (2.43)$$

Taking equation (2.43) a step further by letting R^{-1} be unity, letting k approach infinity and taking the expectation we have,

$$\begin{bmatrix} \hat{a}(k+1|k) \\ \hat{b}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{nu}(k) & -R_{uy}(k-1) \\ -R_{uy}^T(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ R_{yy}(k) \end{bmatrix} \quad (2.44)$$

Note that the time varying measurement vector, $H(k)$, is of the form

$$H(k) = [u(k) u(k-1) \dots u(n-m) | -y(k-1) -y(k-2) \dots -y(k-n)] \quad (2.45)$$

Shown in detail, equation (2.44) has the characteristic form (equation 2.46, next page). It is the Toeplitz and symmetric nature of equation (2.46) that is exploited by the Levinson [Ref. 34] and Lattice [Ref. 35] algorithms.

$$\begin{bmatrix}
R_{uu}(0) & R_{uu}(-1) & \cdots & R_{uu}(-m) \\
R_{uu}(-1) & R_{uu}(0) & \cdots & R_{uu}(1-m) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
R_{uu}(1-m) & \cdots & R_{uu}(0) & -R_{uy}(m-1) \cdots -R_{uy}(m-n) \\
\hline
\hat{\underline{a}}(k+1|k) \\
\hat{\underline{b}}(k+1|k)
\end{bmatrix} = \begin{bmatrix}
-R_{uy}(-1) & -R_{uy}(-2) & \cdots & -R_{uy}(-n) \\
-R_{uy}(0) & -R_{uy}(1) & \cdots & -R_{uy}(1-n) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-R_{yu}(-1) & -R_{yu}(0) & \cdots & -R_{yu}(m-1) \\
-R_{yu}(-2) & -R_{yu}(-1) & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-R_{yu}(-n) & \cdots & \cdots & -R_{yu}(m-n) \\
\hline
R_{yy}(0) & R_{yy}(-1) & \cdots & R_{yy}(-n) \\
R_{yy}(-1) & R_{yy}(0) & \cdots & R_{yy}(1) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
R_{yy}(-n) & \cdots & \cdots & R_{yy}(0) \\
\hline
R_{uy}(0) & R_{uy}(1) & \cdots & R_{uy}(m) \\
R_{uy}(1) & R_{uy}(2) & \cdots & R_{uy}(m-n) \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
R_{uy}(n) & \cdots & \cdots & R_{yy}(n)
\end{bmatrix}^{-1}$$

(2.46)

The Adaptive Kalman ARMA modeling technique can be best visualized by a block diagram. Referring to Figure 2.1, the unknown system is excited by a white, zero mean, gaussian noise sequence of sample values from a random process. The input and its associated output are then passed through M and N delays respectively in serial form. The parallel inputs ($M+1$) and the outputs (N) are concatenated to form the measurement vector $H(k)$ which is represented by equation (2.13). The inputs N and M are selected a-priori as the model orders for the Autoregressive and Moving Average processes one desires to identify. It can be easily seen that the remainder of the figure simply implements equation (2.15a). The outputs of the Adaptive Kalman Identifier are an estimate of the coefficients,

$$\begin{bmatrix} \hat{b}(k|k) \\ \hline \hat{a}(k|k) \end{bmatrix}$$

and a one step prediction, $\hat{y}(k|k)$.

C. OBSERVATIONS

In this section it has been shown that a direct connection can be established between the Adaptive Kalman Identifier and the Yule-Walker equations associated with an AR process. Secondly, the steady state Adaptive Kalman Identifier closely resembles the discrete Weiner-Hopf equation associated with the MA process when the measurement matrix, $H(k)$, is of the form,

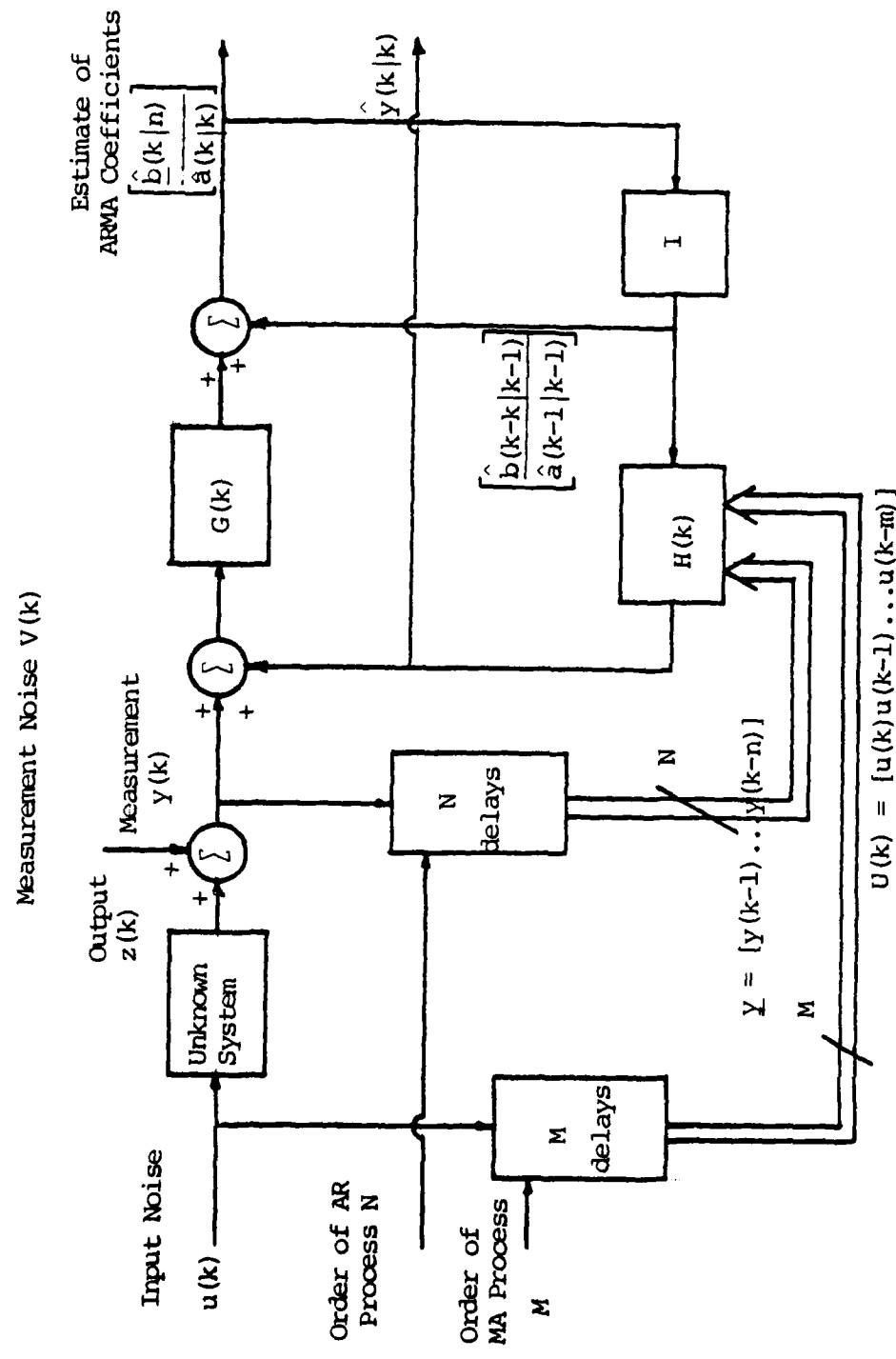


Figure 2.1. Adaptive Kalman Identifier

$$H(k) = [u(k) \ u(k-1) \ u(k-2) \ \dots \ u(k-m)]$$

And, thirdly under the same assumptions as in the previous two cases, the Adaptive Kalman ARMA Identifier is similar to the form Perry [Ref. 6] exploits using Lattice modeling.

III. COMPARISON BETWEEN THE ADAPTIVE MA KALMAN IDENTIFIER AND THE WIDROW LMS ADAPTIVE FILTER

A. PRELIMINARIES

It is instructive to investigate the similarities between the Adaptive Kalman Identifier and the Widrow LMS Adaptive Filter when the concepts are applied to system identification. Basically, the LMS algorithm implementation of system identification considers a block diagram as is shown in Figure 3.1. The output, $y(k)$, of the Adaptive filter is simply a weighted linear combination of the past and present known inputs. The same input is fed into both the unknown system to be identified and the adaptive filter. The output of the unknown system is designated the desired response, $d(k)$, from which an error signal, $\epsilon(k)$, is derived. The error signal, $\epsilon(k)$, provides the criterion through which the weights, w_i , are adjusted such that the error, $\epsilon(k+1)$, is driven toward its minimum. A more detailed analysis of the operation of the LMS algorithm can be found in Ref. 11.

The weights w_i are adjusted from time step to time step in the following manner,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \epsilon(k) \underline{x}(k) \quad (3.1)$$

where we define the following quantities,

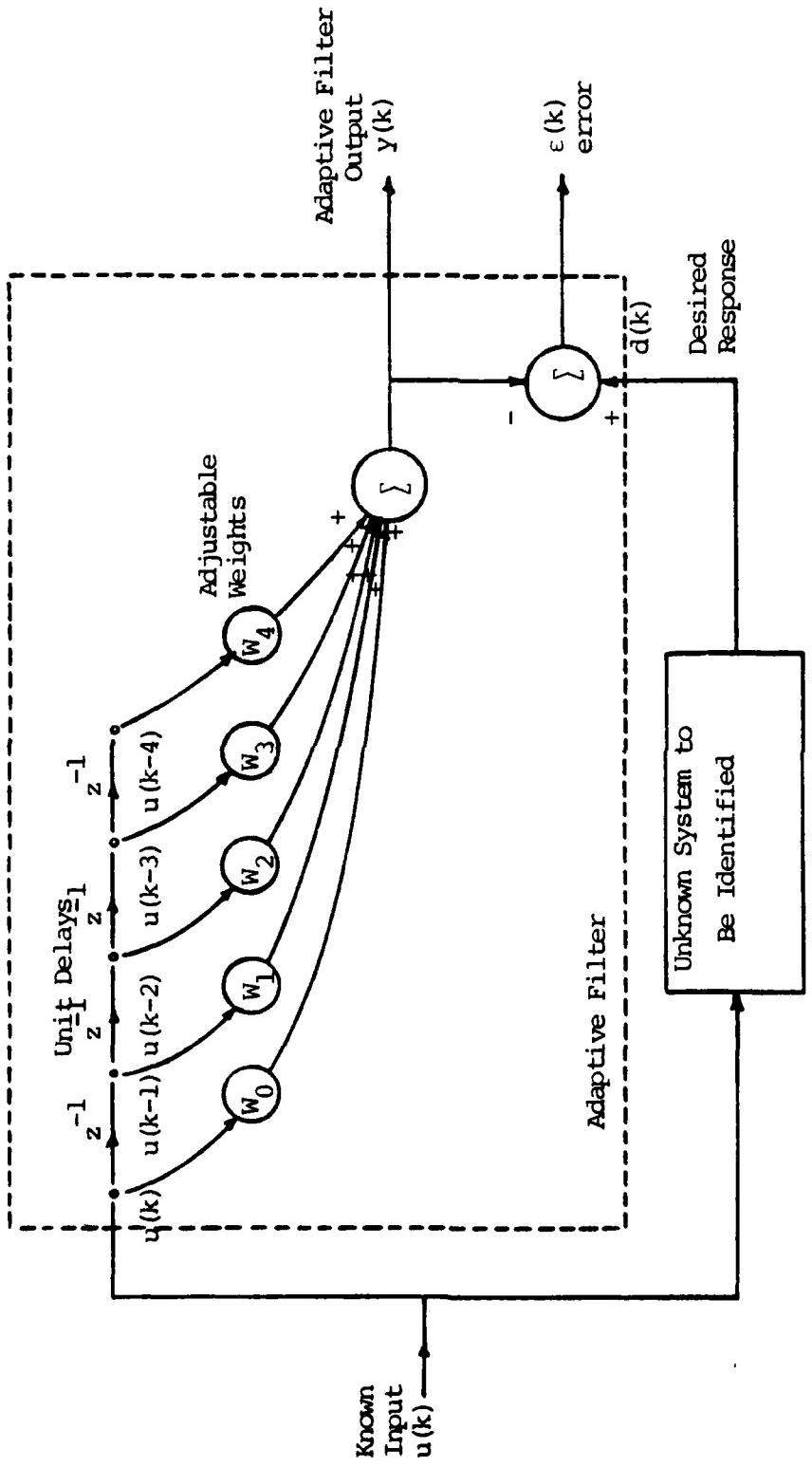


Figure 3.1. LMS Adaptive Filter Implementation of System Identification

$$\underline{w}(k) = \begin{bmatrix} w_0(k) \\ w_1(k) \\ \vdots \\ \vdots \\ w_m(k) \end{bmatrix} \quad (3.2)$$

the weight vector at time step k ,

$$\underline{x}(k) = \begin{bmatrix} u(k) \\ u(k-1) \\ \vdots \\ \vdots \\ u(k-m) \end{bmatrix} \quad (3.3)$$

the input signal vector at time step k ,

$$\varepsilon(k) = d(k) - y(k) \quad (3.4)$$

$$y(k) = \underline{w}^T(k) \underline{x}(k) \quad (3.5)$$

the error, $\varepsilon(k)$, and the output, $y(k)$, at time step k , and, k_s , a scalar constant controlling the rate of convergence and the stability of the adaptive filter.

B. THE COMPARISON

Substituting (3.4) and (3.5) into (3.1) gives,

$$\underline{w}(k+1) = \underline{w}(k) - 2k_s \underline{x}(k) [d(k) - \underline{w}^T(k) \underline{x}(k)]. \quad (3.6)$$

Recalling the MA form of the Adaptive Kalman Identifier, equation (2.33a), rewritten here for convenience as

$$\hat{\underline{a}}(k+1|k) = \hat{\underline{a}}(k|k-1) + K(k) [z(k) - H(k) \hat{\underline{a}}(k|k-1)], \quad (3.7)$$

one can make the following associations:

$$\hat{\underline{a}}(k+1|k) \Leftrightarrow \underline{w}(k+1) \quad (3.8a)$$

$$\hat{\underline{a}}(k|k-1) \Leftrightarrow \underline{w}(k) \quad (3.8b)$$

$$z(k) \Leftrightarrow d(k) \quad (3.8c)$$

$$H^T(k) \Leftrightarrow \underline{X}(k) \quad (3.8d)$$

$$K(k) \Leftrightarrow -2k_s \underline{X}(j) \quad (3.8e)$$

Recall that for the MA form of the AKI, the measurement vector $H(k)$ represents a vector of past and present inputs. Namely,

$$H(k) = [u(k) \ u(k-1) \ \dots \ u(k-m)]. \quad (3.9)$$

Therefore the associations (3.8a)-(3.8d) are straightforward. However, it is not so clear as to what is meant by the association (3.8e). Digressing a moment to present an equivalent expression [Ref. 27] for the Kalman gain, $K(k)$,

$$K(k) = P(k|k) H^T(k) R^{-1} \quad (3.10)$$

and substituting (3.10) into the association (3.8e) and entertain the conjecture that the quantities are equivalent

under certain conditions, we have:

$$P(k|k) H^T(k) R^{-1} = -2k_s \underline{X}(k) . \quad (3.11)$$

The conditions alluded to are (1) that the Adaptive Kalman Identifier is in steady state and (2) that the statistics of the input forcing function are stationary. Denoting the steady state error covariance, $P(k|k)$, as P_∞ , equation (3.11) can be solved for k_s ,

$$k_s I = -\frac{1}{2} R^{-1} P_\infty . \quad (3.12)$$

Invoking the entire Kalman gain history in its more popular form of equation (2.33b) and equating it to the Widrow gain (3.8e), we instead arrive at,

$$-2k_s \underline{X}(k) = P(k|k-1) H^T(k) [H(k) P(k|k-1) H^T(k) + R]^{-1} \quad (3.13a)$$

$$H^T(k) = \underline{X}(k) . \quad (3.13b)$$

Solving for the convergence factor, k_s , we obtain,

$$\begin{aligned} -2k_s \underline{X}(k) \underline{X}^T(k) &= P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k) \\ -2k_s I &= P(k|k-1) \underline{X}(k) [\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1} \underline{X}^T(k) [\underline{X}(k) \underline{X}^T(k)]^{-1}. \end{aligned}$$

But $[\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R]^{-1}$ is a scalar. Therefore,

$$-2k_s I = \frac{P(k|k-1)}{\underline{X}^T(k) P(k|k-1) \underline{X}(k) + R} \quad (3.14)$$

Griffiths [Ref. 22] suggests that the convergence factor k_s , which he denotes as $\mu(n)$, should be chosen such that,

$$\mu(n) = \frac{\alpha}{L \hat{\sigma}_x^2(n)} \quad (3.15)$$

where $0 < \alpha < 1$ is a normalized adaptive stepsize parameter and the term $\hat{\sigma}_x^2(n)$ is an estimate of the input power level which may be computed using a geometrically-weighted average for an L weight adaptive filter,

$$\hat{\sigma}_x^2(n) = (1 - \frac{\alpha}{L}) \hat{\sigma}_x^2(n-1) + \frac{\alpha}{L} x^2(n) . \quad (3.16)$$

Comparing equation (3.14) and (3.15), one observes that if $P(k|k-1)$ is equal to the identity matrix then we obtain a term proportional to the input power. Further if measurement noise is considered negligible, $R \rightarrow 0.0$, then 3.14 and 3.15 are essentially equivalent. The salient point to make, however, is that the choice of the convergence constant, k_s , in the LMS algorithm provides no clue as to how to deal with measurement noise whereas the Adaptive MA Kalman Identifier explicitly incorporates measurement error into its algorithm.

C. OBSERVATIONS

It has been known that the LMS algorithm was suboptimal since the actual gradient of equation (3.1) is replaced by the approximation, $2\varepsilon(k) x(k)$, [Ref. 24]. However, the degree of suboptimality is difficult to quantify. In previous linear prediction research no mention was made regarding the

role of measurement noise. Equation (3.14) gives us some insight into wiser selections of the rate of convergence constant, k_s , since it takes into account the effects of measurement noise. Further, it appears that the LMS adaptive filter is a special case of the Adaptive MA Kalman Identifier.

IV. COMPARISON BETWEEN THE ADAPTIVE ARMA KALMAN IDENTIFIER AND THE ADAPTIVE RECURSIVE LMS FILTER

This chapter endeavors to investigate the similarities and differences between the Adaptive ARMA Kalman Identifier and the Adaptive Recursive LMS Filter [Ref. 18]. LMS filters [Ref. 11] have enjoyed much popularity in the recent past due to their ease of implementation, simple unimodal algorithm, robustness and ability to "adapt" to the unknown statistics of the signal environment. Being transversal in nature, they have a finite impulse response being able to produce only zeros in the input/output transfer function. One may, however, decide to model the transfer function,

$$H_1(z) = \frac{1}{1 - .9z^{-1}} \quad (4.1)$$

using a transversal filter, only to realize that a large number of delays are required in order to arrive at an adequate approximation. The germane point which is being made is that one weighted feedback tap can realize an infinite string of feed-forward coefficients. Moreover, it is very desirous to adjust the feedback weights adaptively in some optimal fashion to the statistics of the signal environment.

A. PRELIMINARIES

The approach taken by Feintuch [Ref. 18] is patterned after the analysis first presented by Widrow [Ref. 11]. A summary of Feintuch's derivation will be presented here with emphasis on its application to ARMA modeling.

Recall the general ARMA equation (2.1b),

$$y(k) = \sum_{i=0}^m a_i u(k-i) + \sum_{i=1}^n b_i y(k-i) \quad (4.2a)$$

which can be rewritten in vector notation,

$$\underline{y}(k) = \underline{a}^T \underline{u} + \underline{b}^T \underline{y} \quad (4.2b)$$

where,

$$\underline{a}^T = [a_0 \ a_1 \ \dots \ a_m] \quad (4.3a)$$

$$\underline{b}^T = [b_1 \ b_2 \ \dots \ b_n] \quad (4.3b)$$

$$\underline{y} = [y(k-1) \ y(k-2) \ \dots \ y(k-n)]^T \quad (4.3c)$$

$$\underline{u} = [u(k) \ u(k-1) \ \dots \ u(k-m)]^T \quad (4.3d)$$

Given that (4.2) is the assumed mathematical description of the unknown system where \underline{u} and \underline{y} , the input and output data sequences, are known, then by solving (4.2) for \underline{a} and \underline{b} , identification of the unknown system can be made. The LMS algorithms, in general, employ a "desired" signal $d(k)$ with which to "train" the adaptive filter. If the desired signal was assumed to be the response of some unknown system to a known input signal, then the algorithm presented by Feintuch [Ref. 18] can be used as a means by which to identify the system parameters.

The first step in the Recursive LMS derivation is to form the error between the desired signal, $d(k)$, and the output

of the filter, $y(k)$,

$$e(k) = d(k) - y(k) = d(k) - \underline{a}^T \underline{u} - \underline{b}^T \underline{y} \quad (4.4)$$

Forming the square of (4.4) and taking the expectation, we have the mean square error representation of the filter,

$$\begin{aligned} E\{e^2(k)\} &= E\{d^2(k)\} + \underline{a}^T R_{uu} \underline{a} + \underline{b}^T R_{yy} \underline{b} \dots \\ &\quad - 2\underline{a}^T R_{du} - 2\underline{b}^T R_{dy} + 2\underline{a}^T R_{uy} \underline{b} \end{aligned} \quad (4.5)$$

where,

$$R_{uu}(k) = E\{\underline{u}(k) \underline{u}^T(k)\} \quad (4.6a)$$

$$R_{yy}(k-1) = E\{\underline{y}(k-1) \underline{y}^T(k-1)\} \quad (4.6b)$$

$$R_{du}(k) = E\{d(k) \underline{u}(k)\} \quad (4.6c)$$

$$R_{dy}(k) = E\{d(k) \underline{y}(k)\} \quad (4.6d)$$

$$R_{uy}(k-1) = E\{\underline{u}(k-1) \underline{y}(k-1)\}. \quad (4.6e)$$

It is assumed that the statistics (4.6) are constants allowing the gradient of (4.5) to be taken with respect to the a_i and b_i . This assumption does not stretch the theory since in practice one uses an input test signal with stationary characteristics and if the unknown system's output process is not stationary, then no identification of the system parameters can be made. The respective gradients are,

$$\frac{\partial}{\partial \underline{a}} [E\{e^2(k)\}] = 2R_{uu}(k)\underline{a} - 2R_{du}(k) + 2R_{uy}(k)\underline{b} \quad (4.7a)$$

$$\frac{\partial}{\partial \underline{b}} [E\{e^2(k)\}] = 2R_{yy}(k)\underline{b} - 2R_{dy}(k) + 2R^T_{uy}(k)\underline{a}. \quad (4.7b)$$

Since the second order statistics are assumed to be known, equations (4.7) can be solved for \underline{a} and \underline{b} by setting the gradients equal to zero. In matrix partitioned form we have,

$$\begin{bmatrix} R_{uu}(k) & | & R_{uy}(k) \\ \hline R^T_{uy}(k) & | & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \hline \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ \hline R_{dy}(k) \end{bmatrix}. \quad (4.8)$$

At this point we digress momentarily to mention that Johnson and Larimore [Ref. 19] and Widrow and McCool [Ref. 20] agree with Feintuch's derivation. The following steps, however, are controversial [Ref. 19,20].

Feintuch continues and states that in general, the statistics involved in (4.8) are not available a-priori. One method of estimating the statistics is to make the filter adaptive in an iterative fashion using the method of steepest descent. The method of steepest descent employs an algorithm of the form,

$$\underline{a}(k+1) = \underline{a}(k) + k_a \frac{\partial}{\partial \underline{a}} [E\{e^2(k)\}] \quad (4.9a)$$

$$\underline{b}(k+1) = \underline{b}(k) + k_b \frac{\partial}{\partial \underline{b}} [E\{e^2(k)\}]. \quad (4.9b)$$

The gradient involved in (4.9) can be approximated using the techniques outlined in [Ref. 11]. The iterative LMS algorithm

for estimating the coefficients \underline{a} and \underline{b} is,

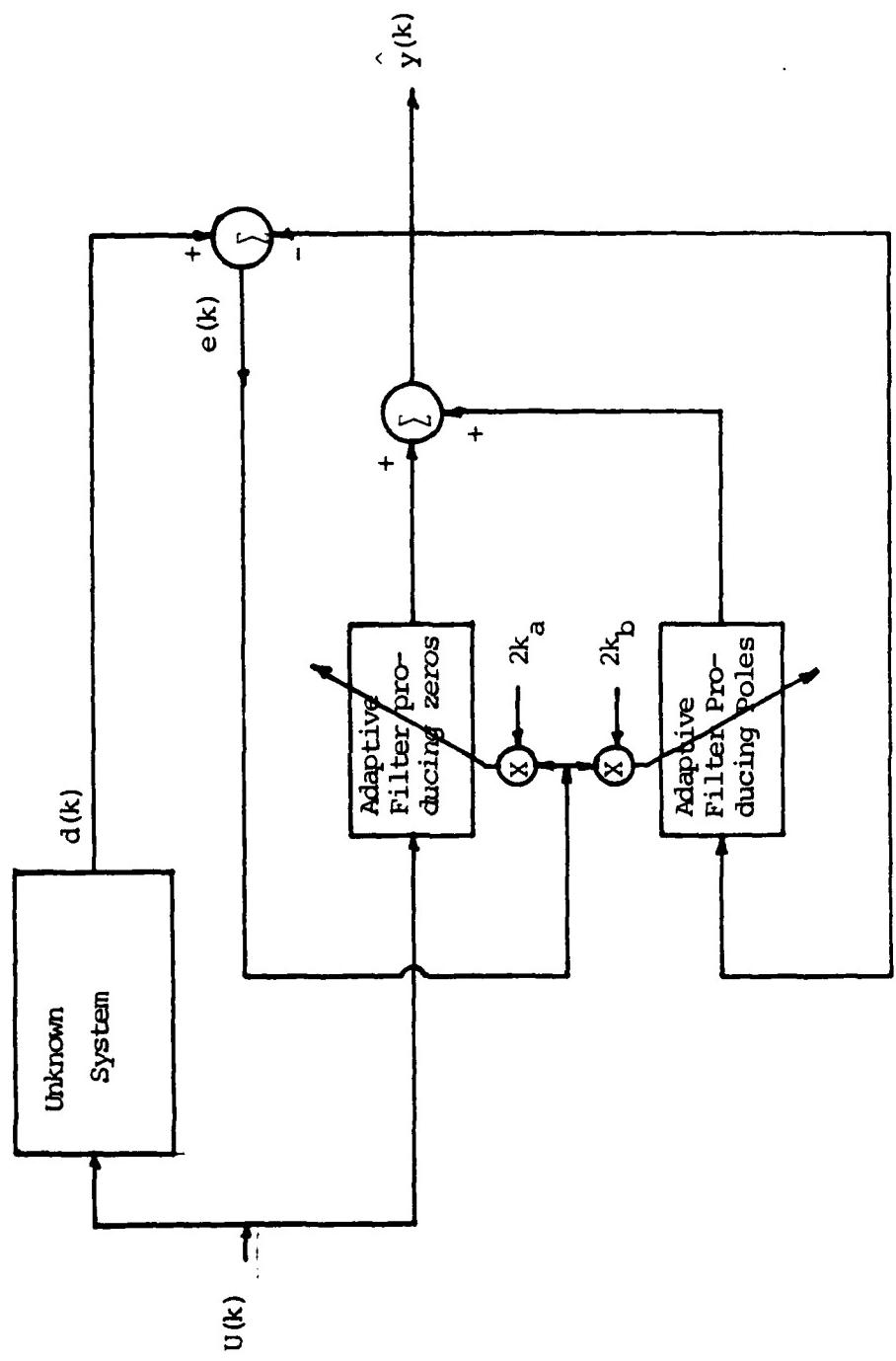
$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.10a)$$

$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \underline{y}(k). \quad (4.10b)$$

Figure 4.1 is a block diagram implementation of the Adaptive Recursive LMS Filter algorithm as applied to system identification. The input signal, $u(k)$, to both the unknown system and the adaptive filter is a zero mean, white gaussian noise sequence of samples from a random process. The output of the unknown system is $d(k)$ which is used as the training signal for the filter. The output of the adaptive filter, $y(k)$, is compared with the desired signal, $d(k)$, from which an error signal is derived. The error signal, $e(k)$, is then used in the LMS algorithm, equation (4.10), to iteratively adjust the \underline{a} and \underline{b} coefficients. Theoretically, when the coefficients of the zeros-producing adaptive filter and the poles-producing adaptive filter correspond exactly with those of the unknown system, the error sequence should be zero and the unknown system is modeled by the Adaptive Recursive LMS Filter. The convergence constants, k_a and k_b , are chosen a-priori as are the initial values, $\underline{a}(0)$ and $\underline{b}(0)$, for the coefficients.

B. THE COMPARISON

One can begin the comparison between the Adaptive Kalman Identifier and the Adaptive Recursive LMS filter by noting



Filter 4.1. Adaptive Recursive LMS Filter Applied to System Identification

the similarities between the equations (2.43) and (4.8), repeated here for convenience,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & -R_{uy}(k-1) \\ -R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{uy}(k) \\ R_{yy}(k) \end{bmatrix} \quad (2.43a)$$

$$A_1 = B_1 C_1 \quad (2.43b)$$

$$\begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix} \begin{bmatrix} \underline{a} \\ \underline{b} \end{bmatrix} = \begin{bmatrix} R_{du}(k) \\ R_{dy}(k) \end{bmatrix} \quad (4.8a)$$

$$\begin{bmatrix} \hat{\underline{a}} \\ \hat{\underline{b}} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & R_{uy}(k-1) \\ R^T_{uy}(k-1) & R_{yy}(k) \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ R_{dy}(k) \end{bmatrix} \quad (4.8b)$$

$$A_2 = B_2 C_2 \quad (4.8c)$$

The upper left partitions of B_1 and B_2 are obviously identical, in that the AKI and the Adaptive Recursive LMS both implement instantaneous estimates of the input covariance functions, $R_{uu}(k)$. The lower right partitions of B_1 and B_2 are similar with one subtle difference. Both are instantaneous covariance functions; however, B_1 employs the covariance, $R_{yy}(k)$, of the actual data whereas B_2 uses, in fact, previous filter estimates to compute the output covariance, $R_{yy}^{(k)}$, more properly denoted by $\hat{R}_{yy}(k)$. The negative sign of the upper right and lower left partitions in B_1 , in comparison with B_2 , is a result of the initial definition for the recursive

weights, b_i . That is, the transfer function for the general ARMA equation can be written as,

$$H_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 - \sum_{i=1}^m b_i z^{-i}} \quad (4.9a)$$

or,

$$H'_{\text{ARMA}}(z) = \frac{a_0 + \sum_{i=1}^m a_i z^{-i}}{1 + \sum_{i=1}^n b_i z^{-i}} \quad (4.9b)$$

The second and more important observation is that cross-correlations employed by equation (4.8b) use the instantaneous values for the past estimates of the outputs while (2.43a) uses the actual past values for $y(k)$. Similarly, comparing C_1 with C_2 , one finds that the lower partition of C_2 uses the cross-correlation between the desired signal and the past estimates of the adaptive recursive LMS filter.

Equation (4.8a) can convey the above information more clearly if it is instead written as,

$$\begin{bmatrix} \hat{a} \\ \cdots \\ \hat{b} \end{bmatrix} = \begin{bmatrix} R_{uu}(k) & | & R_{uy}\hat{(k-1)} \\ \hline & | & \\ R_{uy}^T\hat{(k-1)} & | & R_{yy}\hat{(k)} \end{bmatrix}^{-1} \begin{bmatrix} R_{du}(k) \\ \cdots \\ R_{dy}\hat{(k)} \end{bmatrix} \quad (4.10)$$

The convergence factors, k_a and k_b , can be compared to the steady state Adaptive Kalman Identifier gains by making

the identical analogies (3.8) as was done in the previous chapter. One also observes that since the output data that is processed by the Adaptive Recursive LMS filter are in actuality filter estimates, a more correct version of Feintuch's original algorithm (4.10) is written as,

$$\underline{a}(k+1) = \underline{a}(k) - 2k_a e(k) \underline{u}(k) \quad (4.11a)$$

$$\underline{b}(k+1) = \underline{b}(k) - 2k_b e(k) \hat{\underline{y}}(k) . \quad (4.11b)$$

In augmented form, equation (4.11) is,

$$\begin{aligned} \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} &= \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{u}(k) \\ \hline 2k_b \hat{\underline{y}}(k) \end{bmatrix} [d(k) - \underline{a}(k) \underline{u}(k) - \underline{b}(k) \hat{\underline{y}}(k)] \\ \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} &= \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} - \begin{bmatrix} 2k_a \underline{U}(k) \\ \hline 2k_b \hat{\underline{Y}}(k) \end{bmatrix} \left[d(k) - [\underline{u}(k) \quad \hat{\underline{y}}(k)] \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} \right] . \end{aligned} \quad (4.12)$$

Recalling equation (2.15a) in a slightly different form we have,

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} = \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} + K(k) \begin{bmatrix} z(k) - H(k) \\ \hline \end{bmatrix} \begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} . \quad (4.13)$$

It is a simple matter to explore the similarities between (4.12) and (4.13) and make the following associations,

AKI

Adaptive Recursive LMS

$$\begin{bmatrix} \hat{\underline{a}}(k+1|k) \\ \hline \hat{\underline{b}}(k+1|k) \end{bmatrix} \Leftrightarrow \begin{bmatrix} \underline{a}(k+1) \\ \hline \underline{b}(k+1) \end{bmatrix} \quad (4.14a)$$

$$\begin{bmatrix} \hat{\underline{a}}(k|k-1) \\ \hline \hat{\underline{b}}(k|k-1) \end{bmatrix} \Leftrightarrow \begin{bmatrix} \underline{a}(k) \\ \hline \underline{b}(k) \end{bmatrix} \quad (4.14b)$$

$$z(k) \Leftrightarrow d(k) \quad (4.14c)$$

$$H(k) = [u(k) \dots u(k-m) \dots u(k-n)] \Leftrightarrow [\underline{u}(k) \underline{y}(k)] \quad (4.14d)$$

$$K(k) \Leftrightarrow \begin{bmatrix} 2k_a \underline{u}(k) \\ \hline 2k_b \underline{-}(k) \end{bmatrix} \quad (4.14e)$$

The associations (4.14d) and (4.14e) are the major differences between the AKI and the Adaptive Recursive LMS. It has been shown that the mean square error surface employing not only estimated feed forward coefficients but also estimated feedback (recursive) coefficients is in general multimodal [Ref. 36]. Hence, the Adaptive Recursive LMS algorithm does not minimize the mean square error [Ref. 19,20], and in general the gradient algorithm in this case does not seek the global minimum.

C. OBSERVATIONS

The complex nature of estimating recursive coefficients is yet a formidable problem [Ref. 19,20]. It seems that Feintuch's algorithm is successful due to the a-priori knowledge of the minimal order generating the desired signal. This knowledge is used in setting the order of the adaptive recursive filter.

V. SYNTHETIC DATA GENERATING PLANTS

The Adaptive ARMA Kalman Identifier was tested using computer derived data from several models. The input/output data was collected by driving a known plant with a zero mean, unit variance, white gaussian noise sequence of samples. The AKI algorithm was repeatedly tested using data from progressively more complex models.

A. AUTOREGRESSIVE-MOVING AVERAGE DATA

Autoregressive-Moving Average (ARMA) data was generated using an equation of the form,

$$y(k) = b_1 y(k-1) + b_2 y(k-2) + \dots + b_n y(k-n) \\ + a_0 u(k) + \dots + a_m u(k-m) \quad (5.1)$$

Taking the Z-transform of (5.1) we have,

$$Y(z) = b_1 z^{-1} Y(z) + b_2 z^{-2} Y(z) + \dots + b_n z^{-n} Y(z) \\ + a_0 U(z) + \dots + a_m z^{-m} U(z) \quad (5.2)$$

where $Y(z)$ ($U(z)$) represents the Z-transform of the output (input) and $z^{-n_0} Y(z) (z^{-n_0} U(z))$ represents the Z-transform of the output (input) delayed n_0 time steps. Equation (5.1) has already been defined as the general form of the ARMA process provided that the sequence $u(k-i)$ $i = 0, 1, \dots$ comes from a gaussian random process [Ref. 25]. The ratio,

$$H(z) = \frac{Y(z)}{U(z)} = \frac{a_0 + a_1 z^{-1} + \dots + a_m z^{-m}}{1 - b_1 z^{-1} - \dots - b_n z^{-n}} \quad (5.3)$$

can then be readily recognized as the general transfer function [Ref. 37] of a digital plant. That is, the $u(k)$ is the observed input and the $y(k)$ is the perfectly measured output. Consider that the measurements of the output include some random error, $v(k)$, due to the inaccuracy of the measuring instruments, then the resulting situation is as depicted in Figure 5.1. A practical, judicious and reasonable description of the measurement error is that this be a stochastic process whose distribution is zero mean gaussian. The noisy measurement, $z(k)$, is then the output, $y(k)$, plus the measurement error, $v(k)$.

There are three cases of interest: Case 1: $b_i = 0$ for all i ; Case 2: $a_i = 0$, $i = 1, 2, \dots, m$; Case 3: $a_i \neq 0$, $b_i \neq 0$ for all i .

1. Moving Average Data (Case 1)

Case one is readily recognized as the all zero plant which produces moving average data. A simple second order plant was defined where $a_0 = 1.0$, $a_1 = 2.0$ and $a_2 = 3.0$. The gaussian noise sequences, $u(k)$ and $v(k)$, were obtained using the general purpose IMSL subroutine, GGNML, provided by the computer center at the Naval Postgraduate School.

2. Autoregressive Data (Case 2)

Case two results in the all pole plant producing autoregressive data. The coefficients were selected such that the

plant of Figure 5.1 produced stable data. In precise terms, the poles of the function,

$$H(z) = \frac{1}{1 + \frac{26}{24} z^{-1} + \frac{9}{24} z^{-2} + \frac{1}{24} z^{-3}} \quad (5.4)$$

were located completely within the unit circle in the z-plane ensuring a stable plant [Ref. 37,38]. The noise sequences were obtained as before.

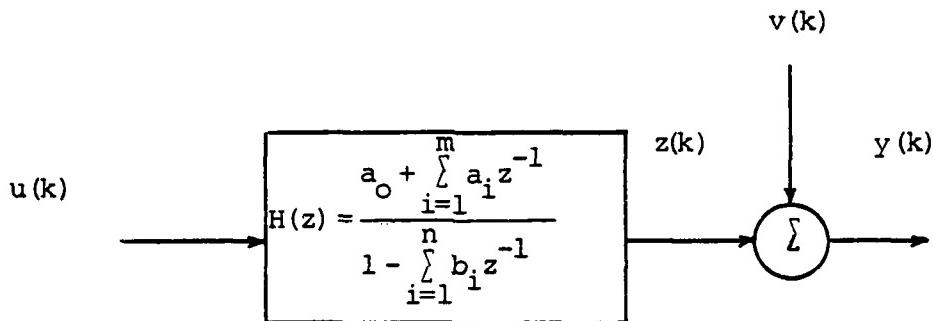


Figure 5.1. ARMA Digital Plant

3. Autoregressive-Moving Average Data (Case 3)

Case three follows from a logical combination of the two previous cases. That is, the transfer function, $H(z)$, now has both zeros and poles. In order to compare with previous work done in the area of system identification [Ref. 6], one of Perry's models [Ref. 6] was used. Namely, the transfer function $H(z)$ used for this case was,

$$H(z) = \frac{1 + 1.4z^{-1} + .98z^{-2}}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \quad (5.5)$$

The input, $u(k)$, was obtained as in the previous two cases using the IMSL subroutine GGNML set to unit variance and zero mean. The output measurement noise sequence, however, was set to a variance of .0001. The basic software, program ARMA, which generated the different data is included as Appendix B. Basically, the coefficients A1-A10 change the character of the general ARMA equation (5.1) which can be selected to produce the desired plant data. The input/output data is then written onto a disk file for subsequent analysis.

In order to compare some of the findings in this thesis with the results previously obtained by Feintuch [Ref. 18], the transfer function,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1.0 - 1.1314z^{-1} + 0.25z^{-2}} \quad (5.6)$$

was also used as a source of synthetic data. Feintuch used (5.6) as a source of data with which to analyze the operation of his Adaptive Recursive LMS Filter.

B. PHASE LOCKED LOOP DATA

The second class of data used to exercise the Adaptive ARMA Kalman Identifier was derived from a computer simulated phase locked loop (PLL) developed by Romeo [Ref. 7]. The basic PLL algorithm, however, implemented a forward Euler integration scheme.

Briefly, in block diagram form, the phase tracking characteristics of the PLL in the frequency domain can be depicted as in Figure 5.2.

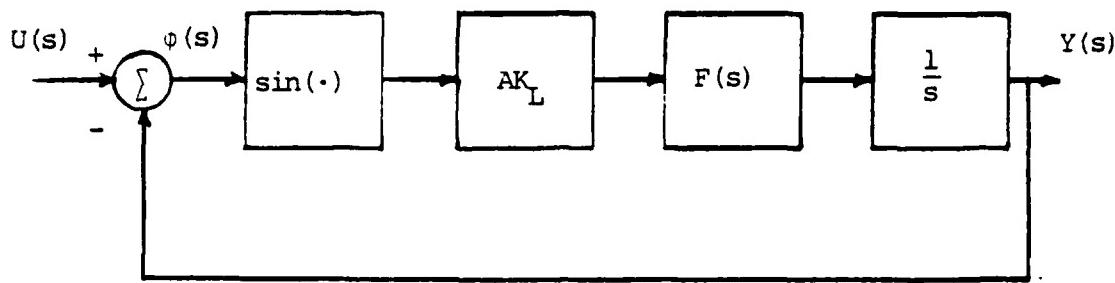


Figure 5.2. PLL Block Diagram

Where:

A is the input voltage

K is the loop gain

$F(s)$ is the filter transfer function

For most applications, it is assumed that $\phi(t) \ll \pi/2$ allowing one to make the approximation, $\sin \alpha \approx \alpha$. This is the linear mode of operation. Romeo [Ref. 7] chose the filter characteristics $F(s) = 1 + K/S$ and the same characteristics were used in this thesis in order to compare results. The parameters of the overall system were adjusted to obtain a damping coefficient, ζ , of 0.3 and a natural frequency, w_n of 3.33 rad/sec. This resulted in a step response overshoot of about 46.7% at $t \approx .75$ sec. Solving for the time domain step response analytically in the linear region one obtains,

$$y(t) = 1 + 1.048e^{-t} \sin(3.178t - 1.266) \quad (5.7)$$

The final block diagram of the PLL system is shown in Figure 5.3.

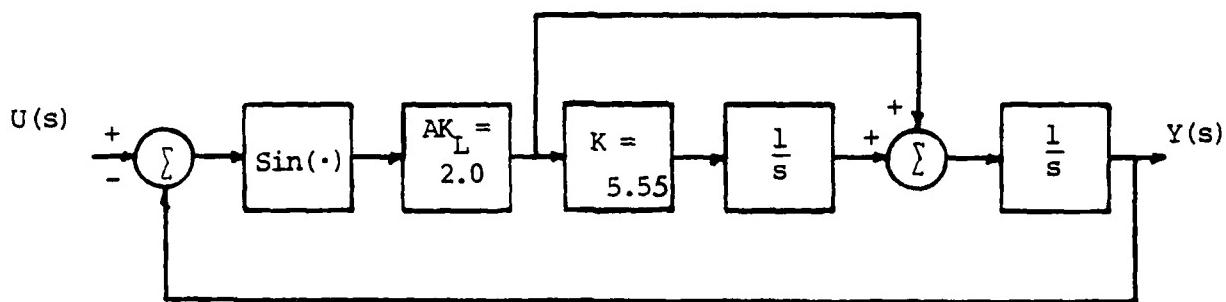


Figure 5.3. Final PLL Block Diagram

Normalized step responses for several step magnitudes using Digital Simulation Language (DSL) were obtained. Figure 5.4 shows the step responses of several step inputs in the range $0 < u(t) < 30$. It is obvious that the PLL is in its approximate linear region and does not exhibit any discernible distortion. Figure 5.5 is a repeat of the above test; however, the range of step inputs are $30 \leq u(t) \leq 170$. It is apparent from Figure 5.5 that the $\sin (\cdot)$ nonlinearity begins to have some effect on the operation of the PLL in that the amplitude of the responses are reduced and delayed.

The DSL PLL system, however, was not used as a source of synthetic data because of the problem of nonstationary statistics of the input signal using the random sequence generators available under DSL already discussed in Ref. 7. These simulations were nevertheless used as a basis for comparison of the operation of the discrete PLL developed in Reference 7 but modified to implement a forward Euler integration scheme which is used in this thesis.

2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION
NORMALIZED STEP RESPONSE .001/.01/.1/1/10/20/30

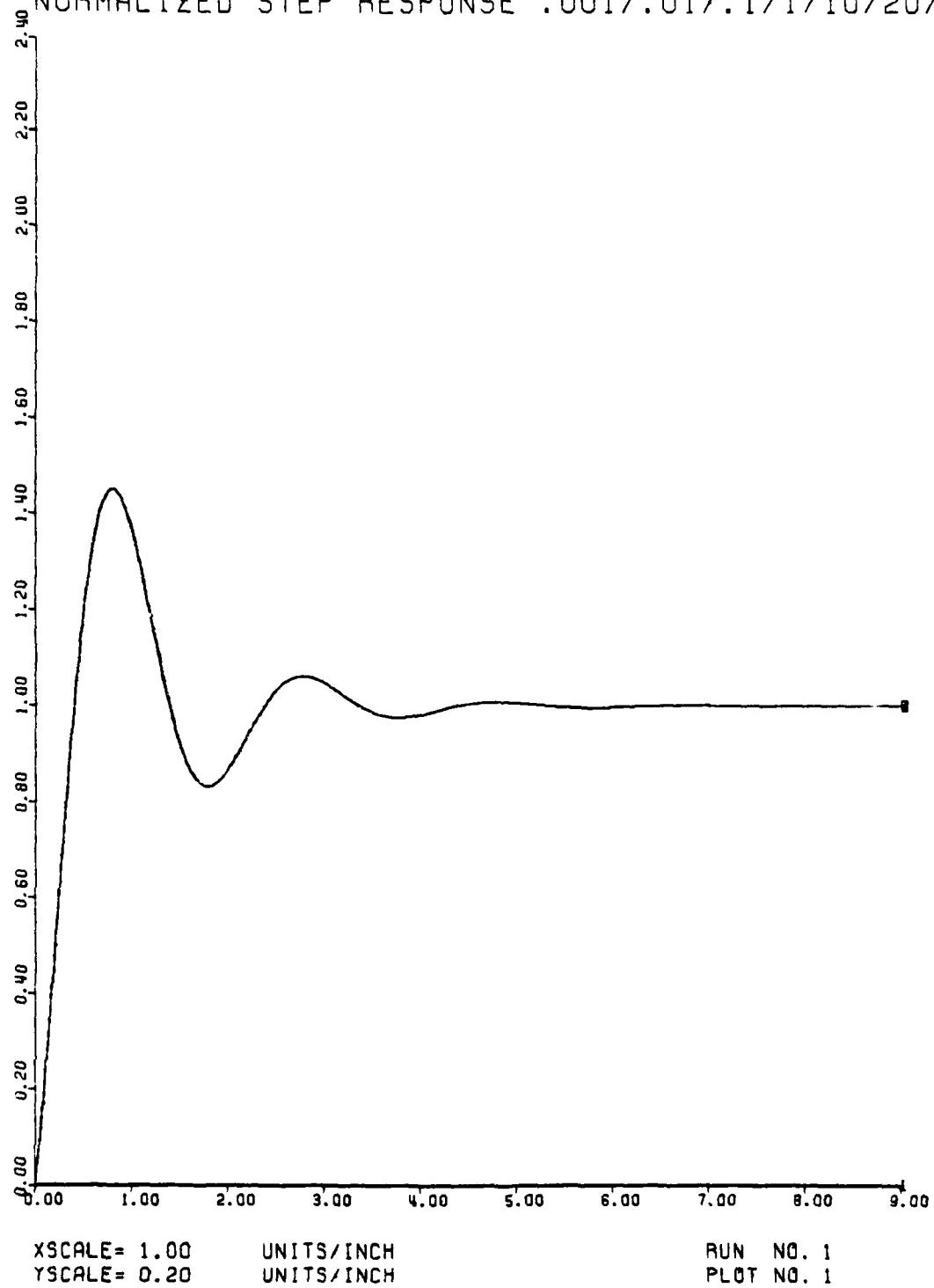


Figure 5.4. PLL Step Response, Linear Region

2ND ORDER PLL -- TRAPEZOIDAL INTEGRATION
NORMALIZED STEP RESPONSE 1/30/60/90/120/150/170

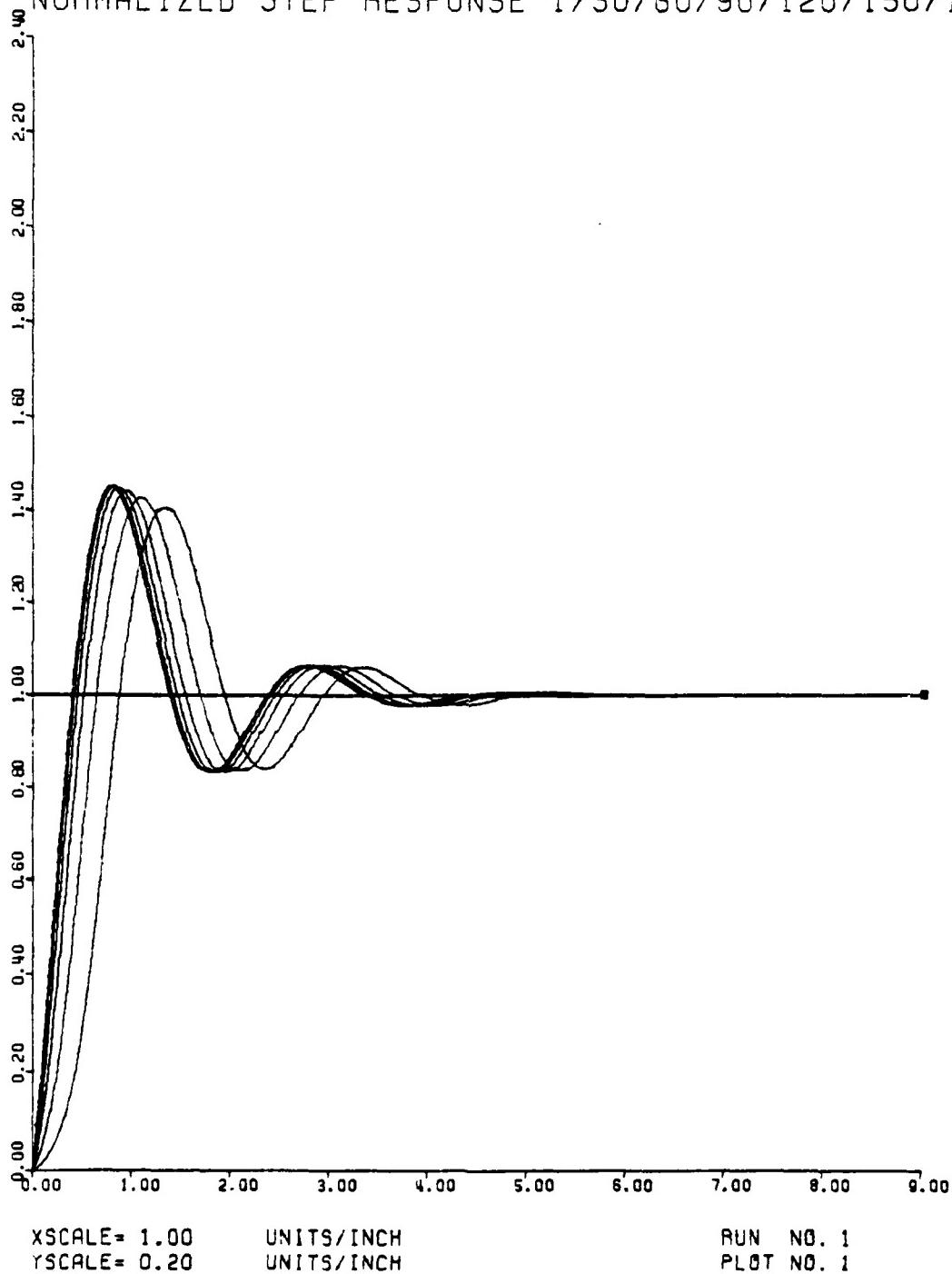


Figure 5.5. PLL Step Response, Non-Linear Region

The forward Euler integrator takes the form,

$$\frac{1}{s} \iff \frac{Tz^{-1}}{1 - z^{-1}}, \quad T = .01 \quad (5.8)$$

The block diagram of Figure 5.3 changes character to appear as in Figure 5.6.

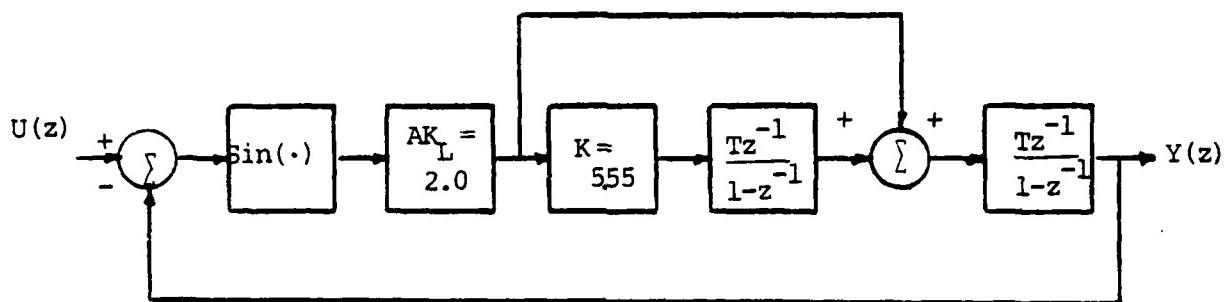


Figure 5.6. Discrete PLL Block Diagram

Applying Mason's gain rule to the block diagram of Figure 5.6 in the linear region we arrive at,

$$\frac{Y(z)}{U(z)} = H(z) = \frac{0.02z^{-1} - 0.018890z^{-2}}{1 - 1.980z^{-1} + .981110z^{-2}} \quad (5.9)$$

the linearized PLL transfer function. The discrete PLL system has zeros at $(0.0, .9445)$ and a complex conjugate pole pair at $(0.990 \pm j.03178)$.

The discrete PLL system was implemented using a Fortran program on the IBM 370 in double precision. The source code for the discrete PLL is included as Appendix C. The discrete

PLL was tested using various magnitude step inputs. Results were similar to those from the DSL simulation. That is, the discrete PLL exhibited the same approximate overshoot, rise time, natural frequency and nearly identical time of max overshoot in the linear region. To implement the PLL using forward Euler integration, the block diagram of Figure 5.6 was reconfigured to appear as Figure 5.7. Note that Figure 5.7 does not employ any delay free loops, hence it is easily programmable.

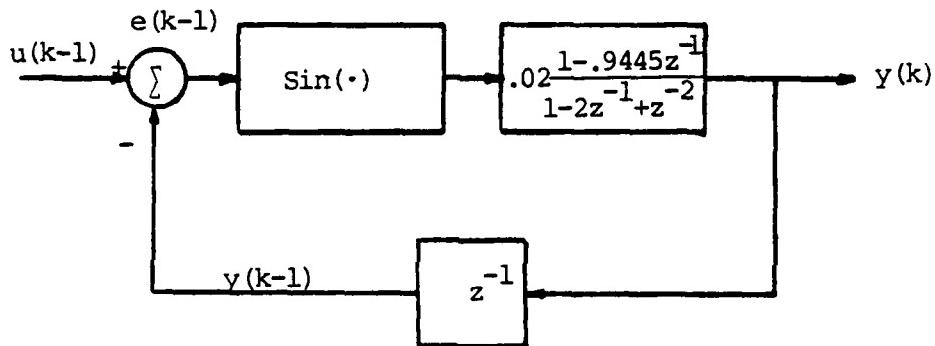


Figure 5.7. Programmable PLL Implementing Forward Euler Integration

Figure 5.8 summarizes the results of the discrete PLL simulation for normalized step inputs in the range $0 < u(t) \leq 30$. Similarly, the discrete PLL was tested in the nonlinear region using normalized step inputs in the range $30 \leq u(t) \leq 170$. The discrete PLL displayed the same characteristics of decreased output amplitude and delay which was first observed in the DSL simulation. Figure 5.9 summarizes the test results of the discrete PLL in the nonlinear region.

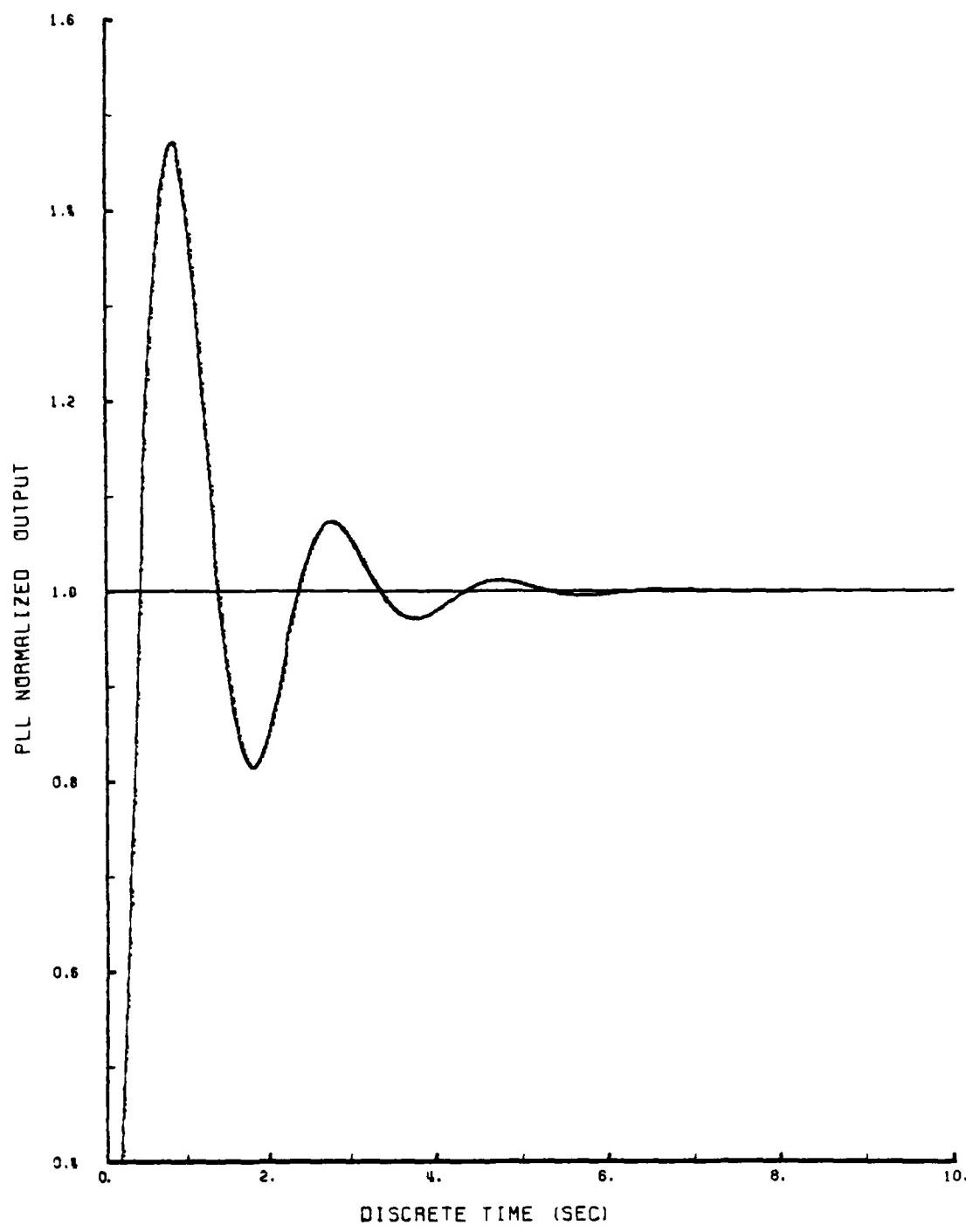


Figure 5.8. Discrete PLL Step Response, Linear Region

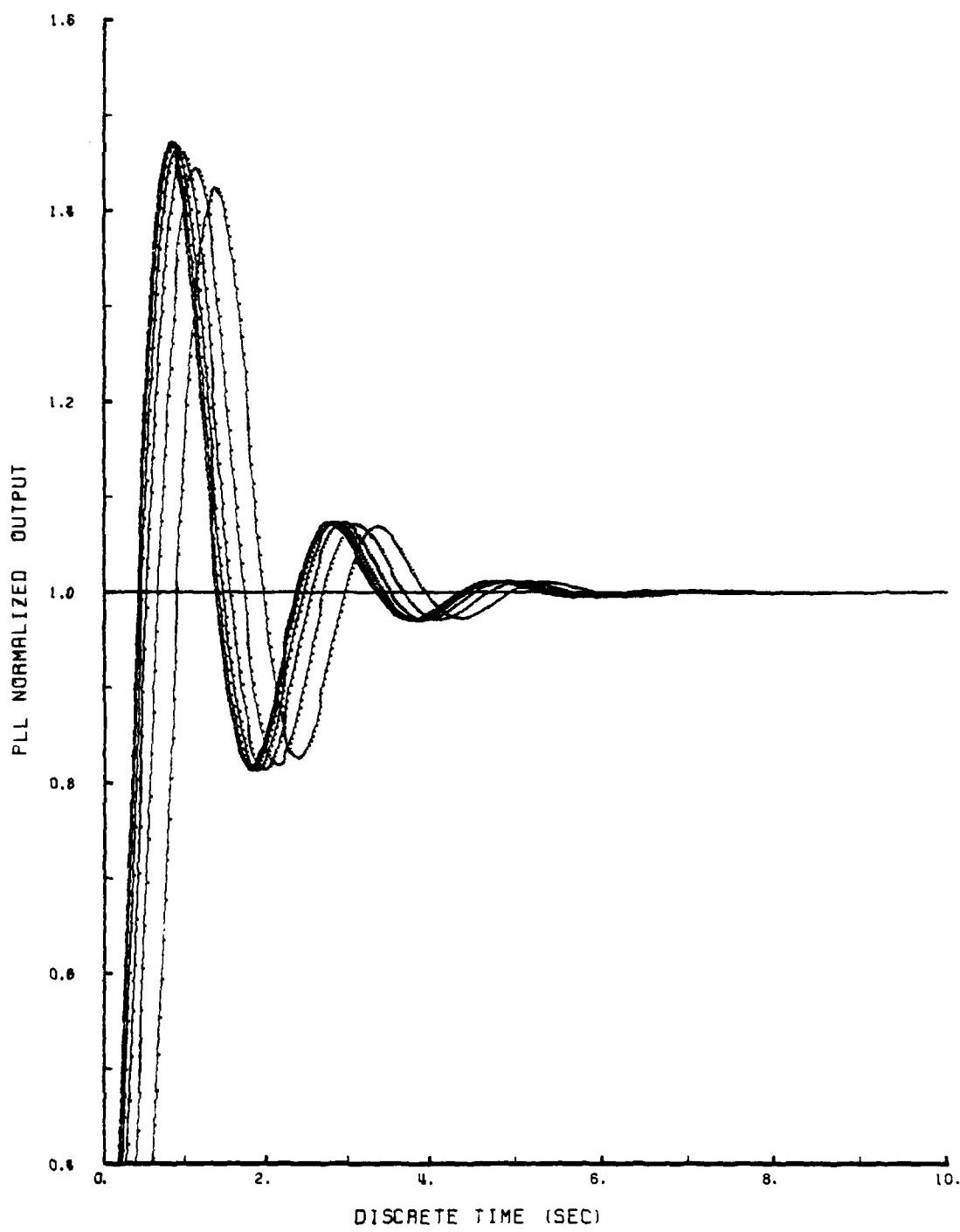


Figure 5.9. Discrete PLL Step Response, Nonlinear Region

It was felt that the PLL was adequately modeled for use as a source of data to be subsequently analyzed by the Adaptive Kalman Identifier.

VI. MODEL IDENTIFICATION SOFTWARE

Three basic Fortran IV programs for use on the IBM 370 were developed:

- (1) ADAPTSN: Adaptive Kalman Identifier
- (2) LMS: Adaptive Least Mean Square Filter
- (3) LMSR: Adaptive Recursive Least Mean Square Filter

All three programs used double precision arithmetic to minimize the effects of truncation error, limit cycles and roundoff error. In all cases the orders of the autoregressive and moving average processes were read in unformatted form from a disk file along with the input/output data of the unknown system to be analyzed. An attempt was made to minimize memory storage, but not at the expense of program flow and clarity.

A. ADAPTIVE KALMAN IDENTIFIER

Program ADAPTSN is a versatile Fortran IV software program which implements equations (2.15) to identify the coefficients of an MA, AR, or ARMA process. Additionally, at each iteration the poles and zeros of the evolving transfer function are computed. ADAPTSN can also be used to perform regression analysis on non-linear terms as discussed in Chapter VII. The versatility of the AKI lies in the various options which are available to the user. By properly selecting the flags N, M, and NL, the user can perform regression analysis on data whose combined order ($N+M+1$) is less than or equal to 20. Table 6.1 lists the ascribed meanings of the various flags.

Table 6.1
AKI FLAG MEANINGS

- N: the order of the autoregressive process
M: one greater than the order of the moving average process
NL = 0: AKI is used to identify the coefficients associated with the general ARMA equation (2.1b) (linear regression)
NL = 1: AKI is used to identify the coefficients of the linear general ARMA equation and the weighting coefficient associated with $u^3(k-i)$
NL = 2: AKI is used as in NL = 0,1 and additionally identifies the weighting coefficient associated with $y^3(k-i)$
NL = 3: AKI is used as in NL = 0,1,2 and additionally identifies the weighting coefficient associated with $u^2(k-i)y(k-i)$
NL = 4: AKI is used as in NL = 0,1,2,3, and additionally identifies the weighting coefficient associated with $y^2(i-i)u(k-i)$
NL = 5: AKI is implemented to analyze time series and identify the ARMA coefficients associated with the Box-Jenkins model [Ref. 26].

The option given by NL = 5 was not extensively tested and as such only limited results are available. Table 6.2 outlines the allowable flag combinations which will produce a valid analysis of the data.

For purposes of this thesis, the non-linear (NL) options were configured to implement the expansion terms associated with the Taylor series expansion of the sine function. This, however, can be changed at the user's discretion to implement any other series expansion by inserting the appropriate Fortran statements at the proper location in the AKI program.

Table 6.2
VALID FLAG COMBINATIONS

OPTION (NL)	MAX COMBINED ORDER
0	$N + M \leq 20$
1	$N + 2M \leq 20; N, M \neq 0$
2	$2N + 2M \leq 20; N, M \neq 0$
3	$3N + 2M \leq 20; N, M \neq 0$
4	$4N + 2M \leq 20; N, M \neq 0$
5	$N + M \leq 20$

The overall structure of the AKI program uses subroutine calls to compute the various quantities necessary for the eventual computation of the system ARMA coefficients. These subroutines are: GAIN, RECKON, PRINT, NEXT. A brief description of the function each performs is given at the beginning of each subroutine. Figure 6.1 describes the general program flow of the AKI. The source code for program ADAPTSN is included as Appendix D.

B. ADAPTIVE LEAST MEAN SQUARE FILTER

Program LMS uses the equations developed in Chapter III to compute the coefficients (or weights) associated with a moving average process. The LMS filter program is capable of computing up to 12 MA coefficients; or equivalently, is capable of identifying an eleventh order moving average process. The general program flow of the LMS filter is presented in Figure 6.2 and the annotated source code is included as Appendix E.

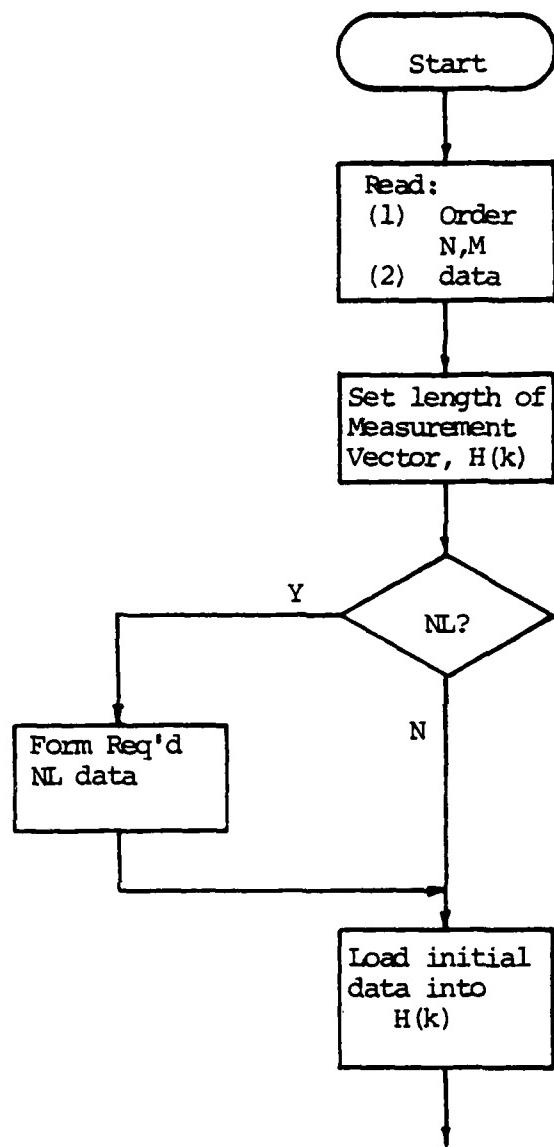


Figure 6.1. AKI General Program Flow

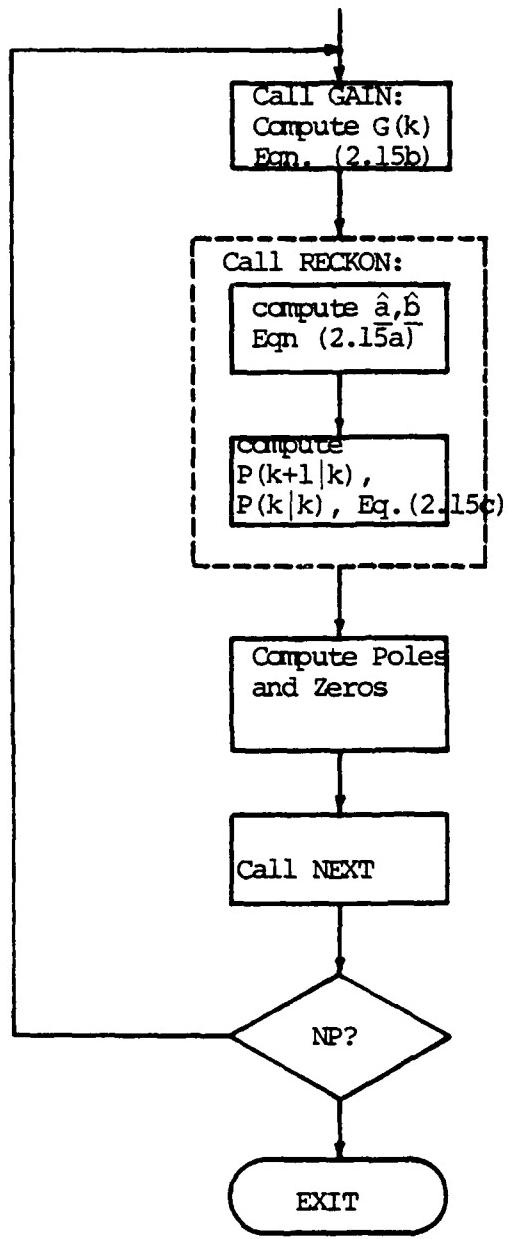


Figure 6.1. (CONTINUED)

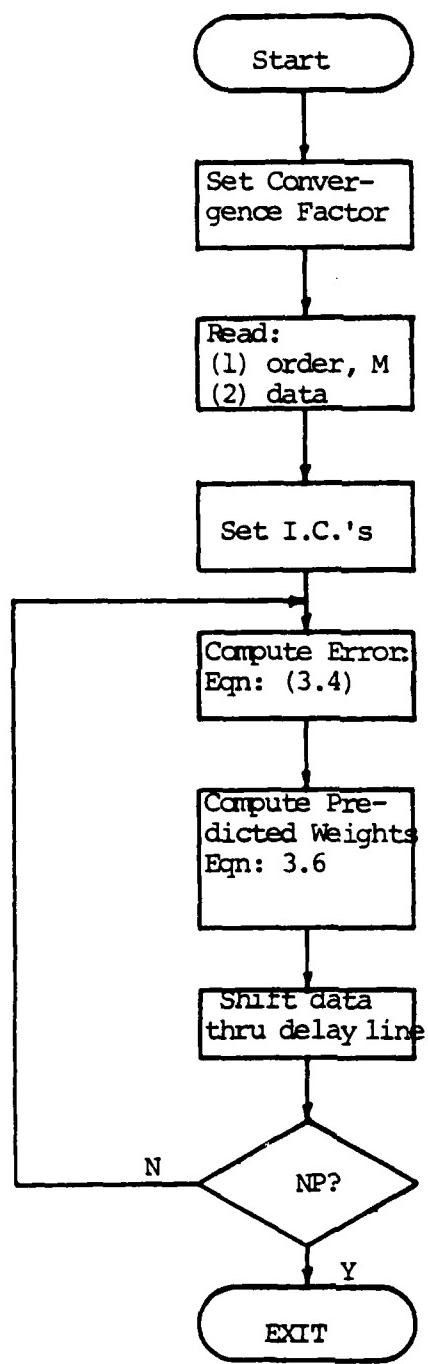


Figure 6.2. LMS Filter Program Flow

C. ADAPTIVE RECURSIVE LEAST MEAN SQUARE FILTER

Program LMSR realizes the algorithm proposed by Feintuch and recapped in Chapter IV. It was designed to handle a combined AR and MA order of 11. That is, the program can be used to estimate $m + 1 = M$ coefficients associated with the MA process and $n = N$ coefficients associated with the AR process such that $N + M \leq 12$. The general program flow for the Adaptive Recursive LMS filter is shown in Figure 6.3 and the source code is included as Appendix F.

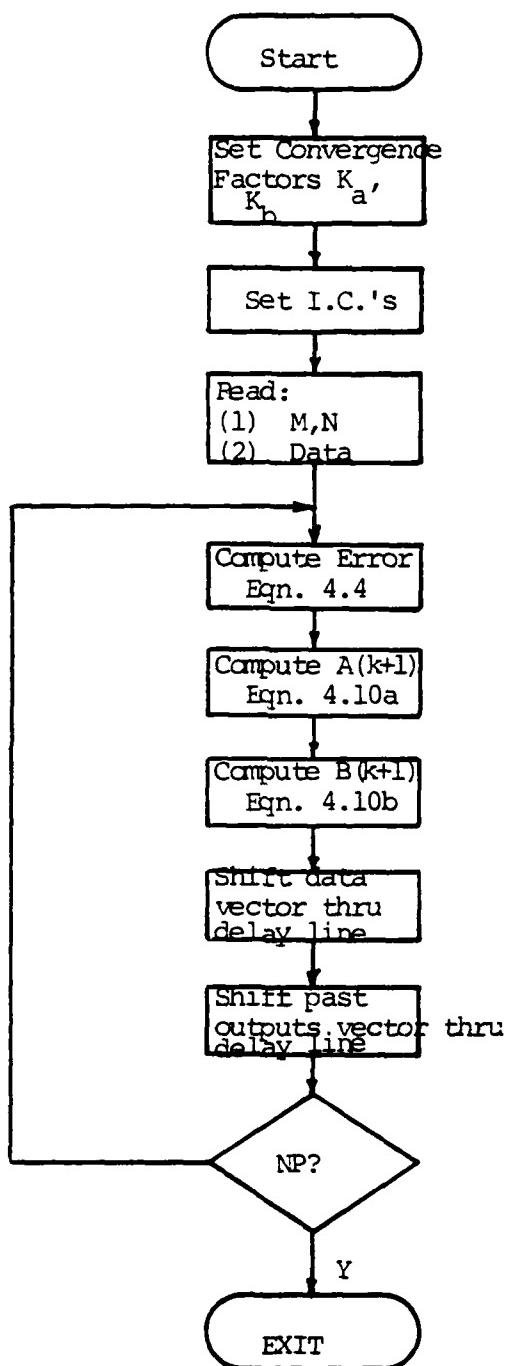


Figure 6.3. LMSR Filter Program Flow

VII. NON-LINEAR IDENTIFICATION

The Adaptive Kalman Identifier can be modified to estimate the weighting coefficients associated with higher order terms produced by some non-linearity within the unknown system. In general, much has to be known about the non-linearity such that the functional description chosen for it is a close approximation to the effects it causes. In this thesis one possible application of the Adaptive Kalman Identifier toward identifying a system with a known non-linearity is explored. No attempt has been made to present an extensive treatment of non-linear analysis techniques.

The approach taken has been previously explored by Parker [Ref. 8]. A special case of the generalized non-linear ARMA model [Ref. 8],

$$\begin{aligned}
 y(k) = & \sum_{i_1=0}^{\infty} a(i_1)u(k-i_1) + \sum_{i_1=0}^{\infty} \sum_{i_2=0}^{\infty} a(i_1, i_2)u(k-i_1)u(k-i_2) + \dots \\
 & + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} a(i_1, \dots, i_m)u(k-i_1) \dots u(k-i_m) \\
 & + \sum_{j_1=1}^{\infty} b(j_1)y(k-j_1) + \sum_{j_1=1}^{\infty} \sum_{j_2=1}^{\infty} b(j_1, j_2)y(k-j_1)y(k-j_2) + \dots \\
 & + \sum_{j_1=1}^{\infty} \dots \sum_{j_n=1}^{\infty} b(j_1, j_2, \dots, j_n)y(k-j_1)y(k-j_2) \dots y(k-j_n) \\
 & + \sum_{i_j=0}^{\infty} \sum_{j_1=1}^{\infty} c(i_j, j_1)u(k-i_j)y(k-j_1) + \dots + \sum_{i_1=0}^{\infty} \dots \sum_{i_m=0}^{\infty} \sum_{j_1=1}^{\infty} \\
 & \dots \sum_{j_n=1}^{\infty} c(i_1, \dots, i_m, j_1, \dots, j_n)u(k-i_1)\dots u(k-i_m)y(k-j_1)\dots y(k-j_n)
 \end{aligned} \tag{7.1}$$

(7.1)

is used to model the input/output relationships, non-linear transfer function, of the phase locked loop. The non-linear element, the sine function, of the PLL can be replaced by a Taylor power series expansion,

$$\sin(x) = x - \frac{1}{3!} x^3 + \frac{1}{5!} x^5 - \dots \quad (7.2)$$

Other expansions can be used, e.g., Legendre polynomials, Volterra series, ..., etc.; however, only the Taylor expansion was investigated in this thesis. Practical implementation of (7.2) suggests truncation at the third order term. The sine function is therefore approximated as

$$\sin(x) = x - \frac{1}{3!} x^3 . \quad (7.3)$$

Substituting (7.3) into the block diagram of Figure (5.6) we have Figure 7.1.

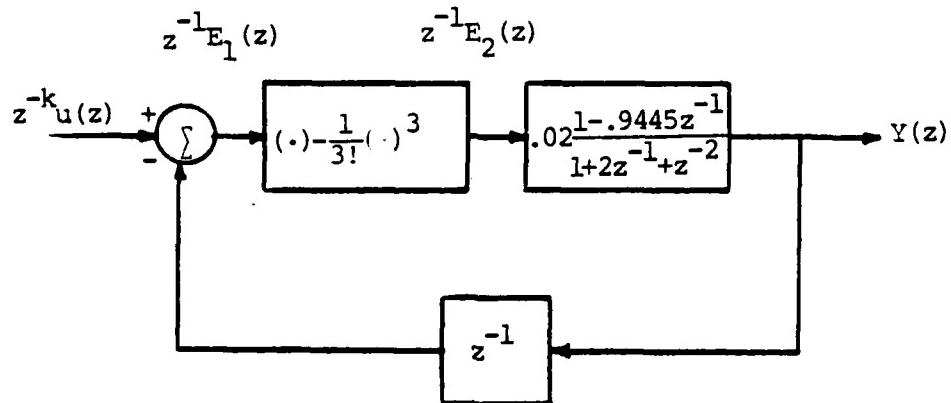


Figure 7.1. PLL, Third Order Taylor Series Approximation

The discrete time domain equations at the different nodes are:

$$e_1(k-1) = u(k-1) - y(k-1) \quad (7.4a)$$

$$e_2(k-1) = e_1(k-1) - \frac{1}{3!} e_1^3(k-1) \quad (7.4b)$$

$$\begin{aligned} y(k) = & .02e_2(k-1) - .01889e_2(k-2) + 2.0y(k-1) \\ & - y(k-2) \end{aligned} \quad (7.4c)$$

Manipulating equations (7.4a)-(7.4c) gives

$$\begin{aligned} y(k) = & .02u(k-1) - .01889u(k-2) + 1.98y(k-1) - .98111y(k-2) \\ & - .003333u^3(k-1) + .003148333u^3(k-2) \\ & + .003333y^3(k-1) - .003148333y^3(k-2) \\ & + .01u^2(k-1)y(k-1) - .09445u^2(k-2)y(k-2) \\ & - .01u(k-1)y^2(k-1) + .09445u(k-2)y^2(k-2). \end{aligned} \quad (7.5)$$

Equation (7.5) indicates which non-linear terms of equation (7.1) should be retained. Therefore, the third order non-linear approximation model should contain the following terms:

$$u(k-1), u(k-2), y(k-1), y(k-2) \quad \text{linear terms} \quad (7.6a)$$

$$u^3(k-1), u^3(k-2) \quad \text{input cubic terms} \quad (7.6b)$$

$$y^3(k-1), y^3(k-2) \quad \text{output cubic terms} \quad (7.6c)$$

$$\left. \begin{array}{l} u^2(k-1)y(k-1), u^2(k-2)y(k-2) \\ y^2(k-1)u(k-1), y^2(k-2)u(k-2) \end{array} \right\} \text{third order cross product terms} \quad (7.6d)$$

The AKI algorithm is primarily modified to include the hybrid signals (7.6) in the structure of $H(k)$, the measurement vector. Since now the measurement vector, $H(k)$ takes the form,

$$\begin{aligned} H(k) = & [u(k) \dots u(k-m), -y(k-1) \dots -y(k-n), u^3(k) \dots \\ & \dots u^3(k-m), -y^3(k-1) \dots -y^3(k-n), -(u^2(k-1)y(k-1)) \dots \\ & \dots -(u^2(k-n)y(k-n)), (u(k-1)y^2(k-1)) \dots (u(k-n)y^2(k-n))] \end{aligned} \quad (7.7)$$

the AKI algorithm calculates the coefficients associated with the special case of the generalized non-linear ARMA model, equation (7.1).

Romeo using a similar approach [Ref. 7] computes a least squares curve fit for the third order truncation of the Taylor series for the sine function over the interval $(0, \pi/2)$. Beginning with the approximation,

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.8)$$

Romeo finds the values for α and β to be .865 and -.095 respectively. Performing the same operations as those used in obtaining equation (7.5) we arrive at,

$$\begin{aligned}
y(k) = & .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\
& + (.01889\alpha - 1)y(k-2) - .02\beta u^3(k-1) + .01889\beta u^3(k-2) \\
& - .01889\beta y^3(k-2) + .06\beta u^2(k-1)y(k-1) + .02\beta y^3(k-1) \\
& - .056670\beta u^2(k-2)y(k-2) - .06\beta y^2(k-1)u(k-1) \\
& + .056670\beta y^2(k-2)u(k-2)
\end{aligned} \tag{7.9}$$

Table 7.1 tabulates the resulting weighting coefficients using the three methods just described: (1) analytically calculated values using the truncated Taylor series, (2) least squares estimate of the third order sine approximation, as computed by Romeo [Ref. 7] and (3) the coefficient estimates as computed using the AKI algorithm.

The tabulation clearly shows that the AKI outperforms the first two methods overall. That is, the linear terms were identified without question; however, the AKI failed to identify the coefficients associated with the $y^3(i-1)$, $y^3(k-2)$, $u^2(k-2)$ $y(k-2)$ and $y^2(k-2)$ $u(k-2)$ terms. The reason for the failure is not known and was not investigated.

Table 7.1
NON-LINEAR WEIGHTING COEFFICIENTS CALCULATED
USING THREE METHODS

(1)	(2)	(3)
TAYLOR SERIES	LEAST SQUARES EST.	AKI USING ACTUAL
$\sin(x) = x - \frac{1}{3!}x^3$	$\sin(x) = \alpha x + \beta x^3$ $\alpha = .865, \beta = -.095$	$\sin(x)$ data base
y(k-1)	1.98	1.980
y(k-2)	-.98111	-.9815
u(k)	0.0	.00003691
u(k-1)	0.2	.02003
u(k-2)	-.01889	-.01892
$u^3(k)$	0.0	.00001471
$u^3(k-1)$	-.003333	-.003278
$u^3(k-2)$.003148333	.003032
$y^3(k-1)$.003333	-.05695
$y^3(k-2)$	-.003148333	.05799
$u(k-1)y(k-1)$.01	.01032
$u^2(k-2)y(k-2)$	-.09445	-.009610
$y^2(k-1)y(k-1)$	-.01	-.01640
$y^2(k-2)u(k-2)$.09445	.01913

VIII. FINDINGS AND CONCLUSION

The performance of the Adaptive Kalman Identifier was compared to the LMS Adaptive Filter and the Adaptive Recursive LMS Filter using data derived from the models discussed in Chapter V. Results for the case where the order of the model (that is, m and n of equation (2.1b)) are assumed known are presented first followed by the analysis of the PLL data. The findings for the overmodeled case are presented next. The graphs show typical runs and do not represent ensemble averages.

A. ORDER OF THE UNKNOWN SYSTEM IS KNOWN

1. AKI vs. LMS Adaptive Filter

Using the MA form of the Adaptive Kalman Identifier, its performance can be compared against that of the LMS adaptive filter. Synthetic data derived from a plant whose transfer function is,

$$H(z) = 1.0 + 2.0z^{-1} + 3.0z^{-2} \quad (8.1)$$

was used. However, a fair comparison necessitated that the LMS Adaptive filter be "tuned" by adjustment of the convergence factor, k_s . Several values for k_s in the range [-.600, -.200] were used. The objective of tuning the LMS filter was to achieve a fast convergence time with little or no steady state error while not compromising filter stability. The three filter weights were normalized and plotted for five

convergence factor values. It can be seen from Figures 8.1-8.3 that convergence is essentially reached by step thirty-five, ($k \approx 35$). As the convergence factor is further increased it can be noted in Figure 8.4 that the filter weights become more noisy, and that when $k_s = -.600$, filter stability is being compromised (Figure 8.5a).

Using the same data, the AKI converges to the MA coefficients in less than five iterations. Referring to Figure 8.5b, it was also noted that as the measurement noise $v(k)$ was increased, the LMS algorithm yielded more noisy estimates whereas the AKI tended to compensate for measurement noise. This is intuitively reasonable since the AKI incorporates into its algorithm the effects of measurement error due to noisy sensors. Comparing Figure 8.6 with Figure 8.7 the latter figure clearly shows the faster convergence of the AKI to the plant coefficient.

It was shown in Chapter III that the AKI gains would approach a steady state value and indeed they did, as Figure 8.8 shows. However, the instantaneous Kalman gains display a more erratic pattern as shown in Figure 8.9. The information of Figure 8.8 was gleaned from Figure 8.9 by computing the average of the individual gains at each iteration using the algorithm,

$$E\{\underline{G}(k+1)\} = E\{\underline{G}(k)\} + \frac{1}{k+1} [\underline{G}(k) - E\{\underline{G}(k)\}] \quad (8.2)$$

where,

$E\{\underline{G}(k+1)\}$ is the one step prediction of the average value for the gains,

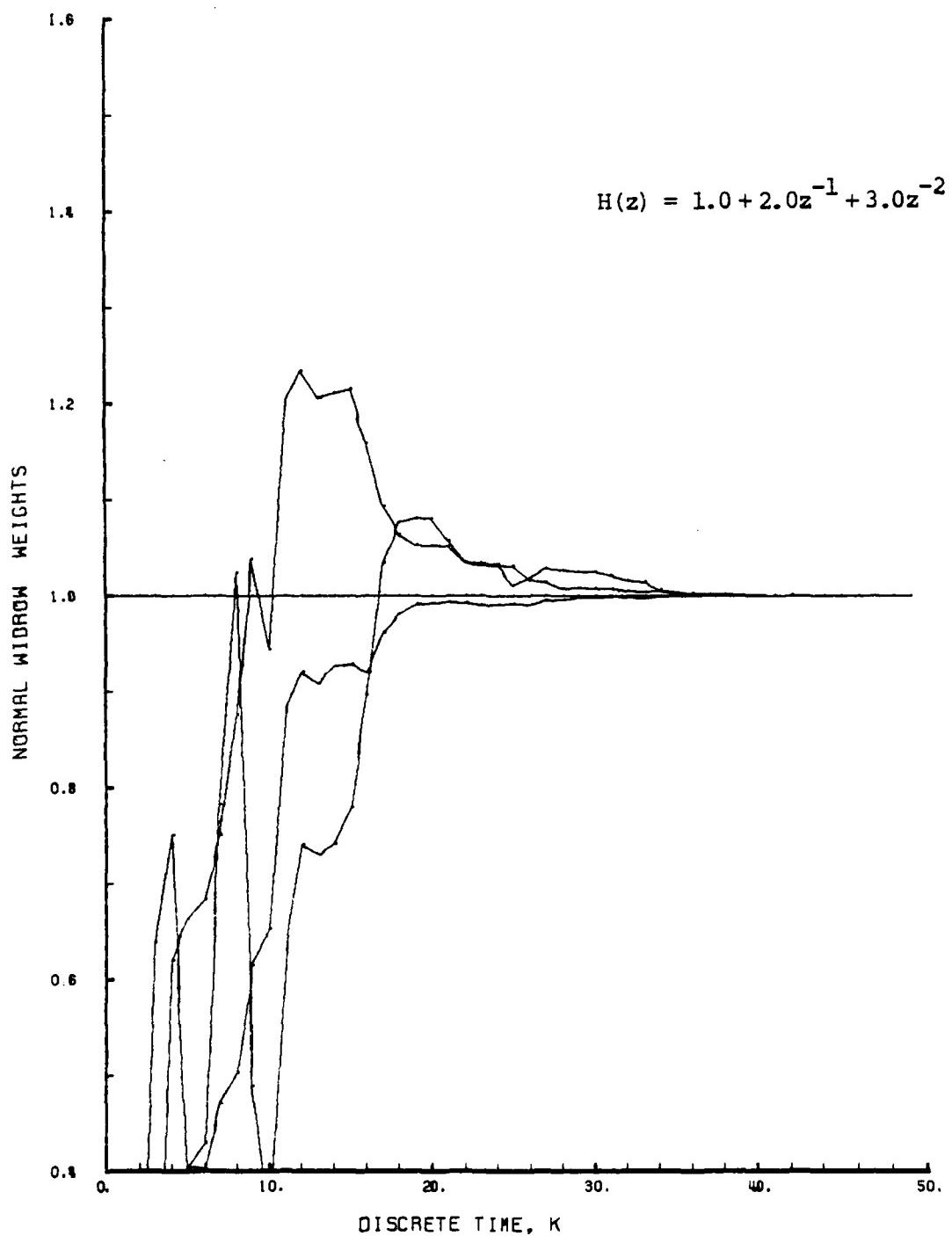


Figure 8.1. LMS Adaptive Filter Weights, $k_s = -.200$

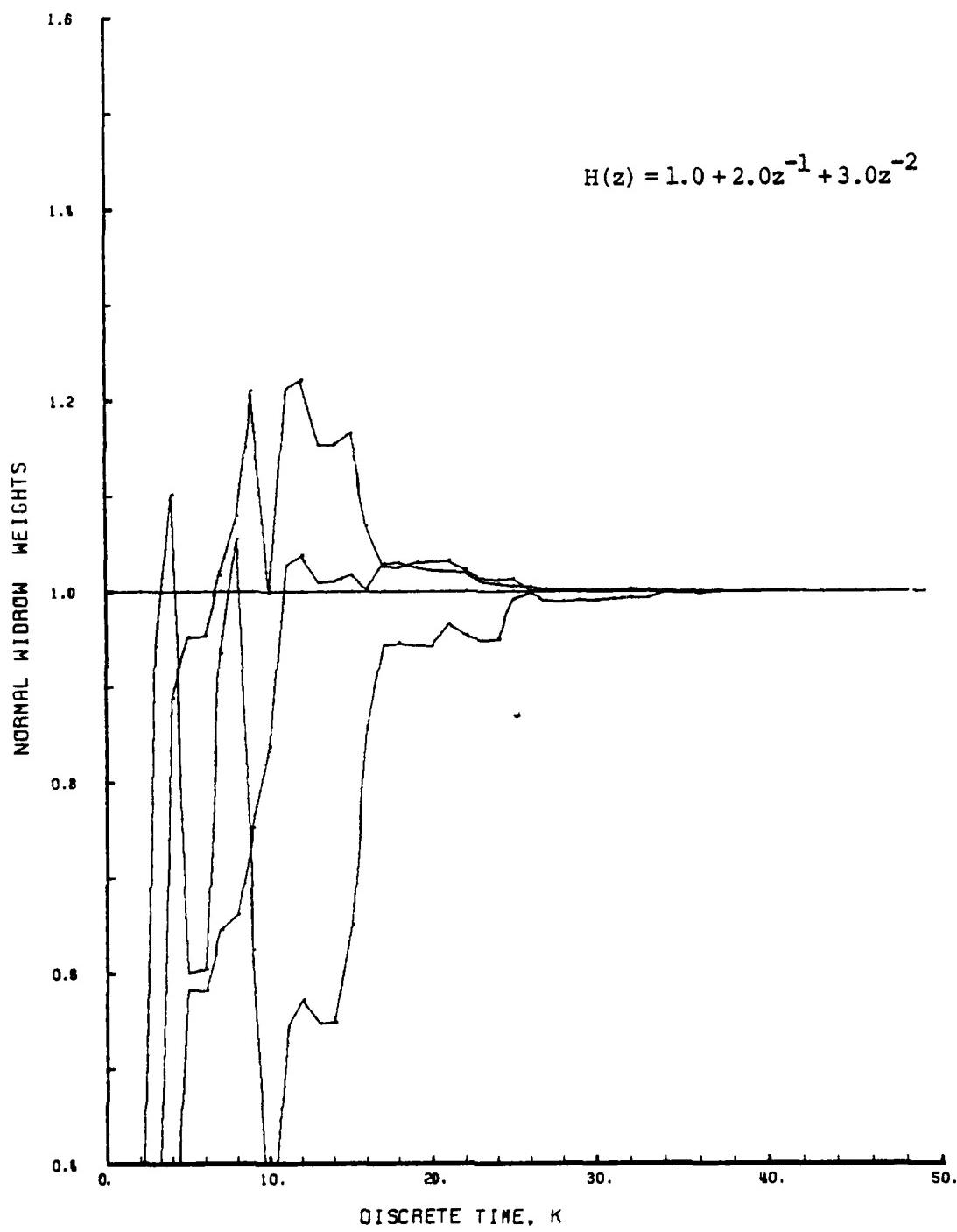


Figure 8.2. LMS Adaptive Filter Weights, $k_s = -.300$

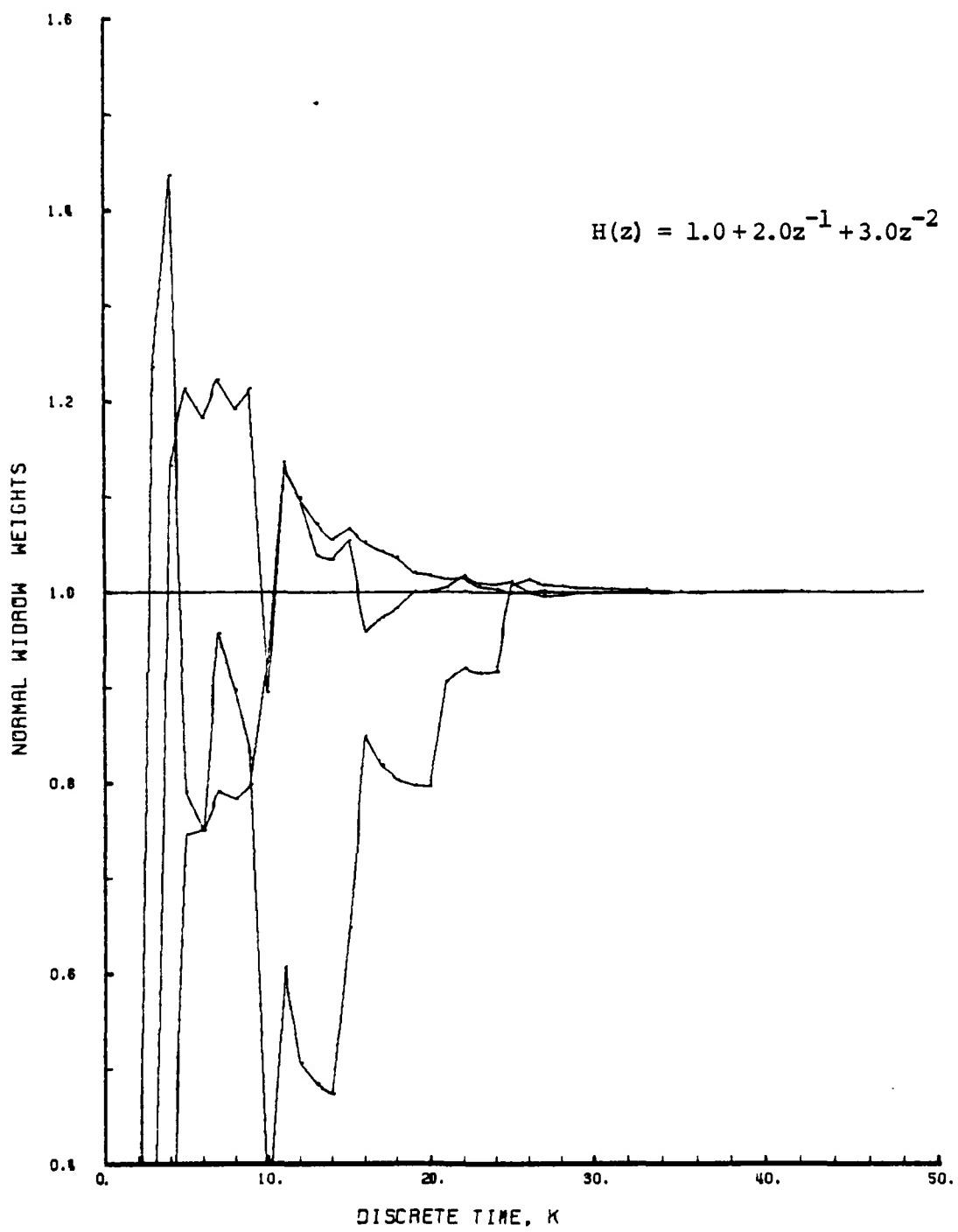


Figure 8.3. LMS Adaptive Filter Weights, $k_s = -.400$

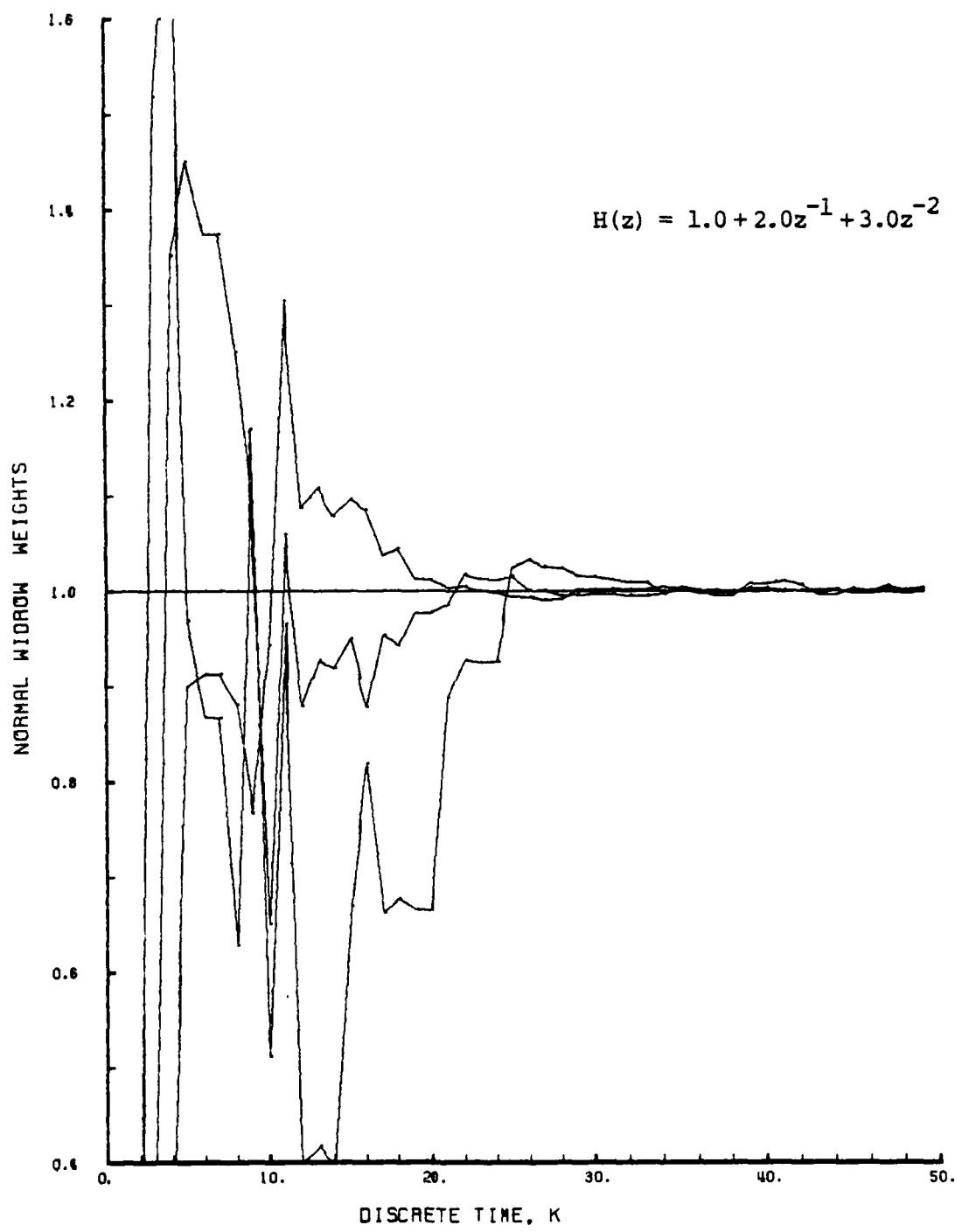


Figure 8.4. LMS Adaptive Filter Weights, $k_s = -.500$

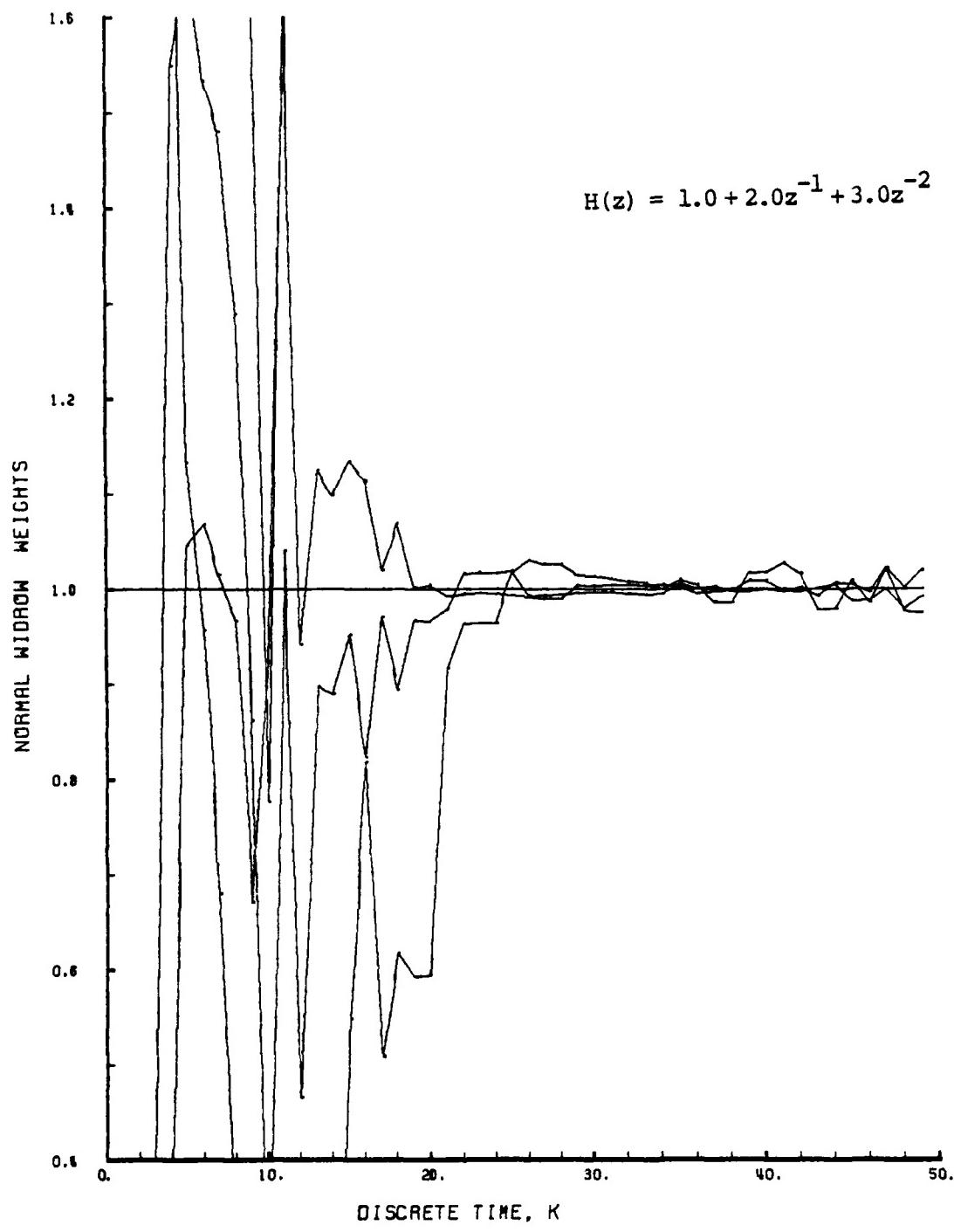


Figure 8.5a. LMS Adaptive Filter Weights, $k_s = -.600$

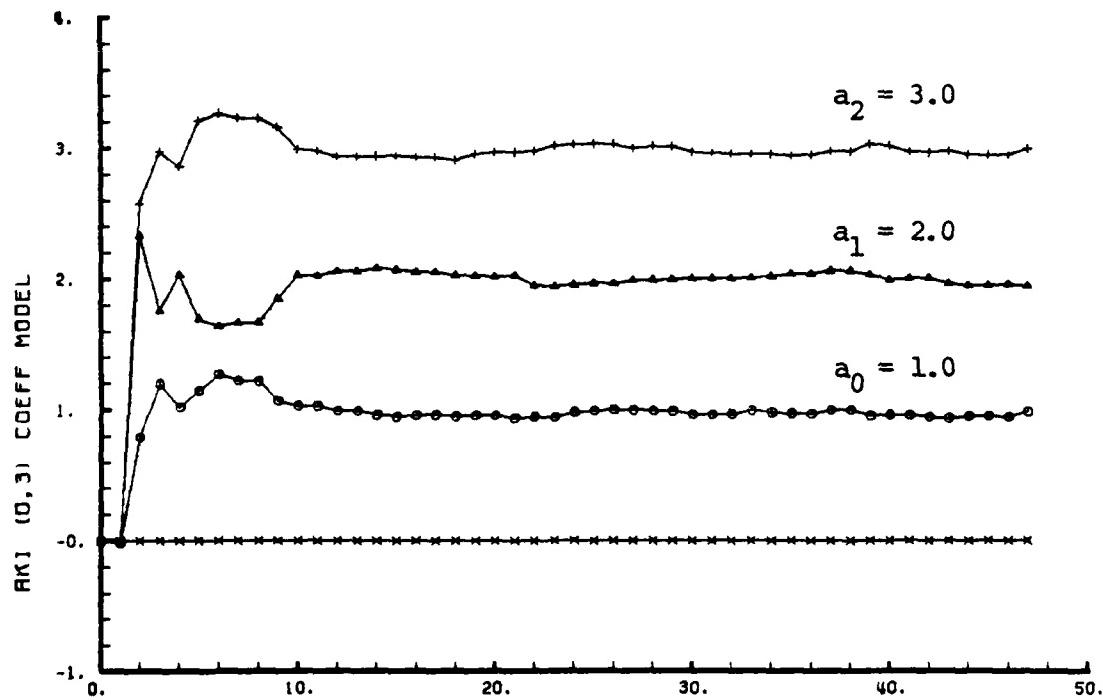
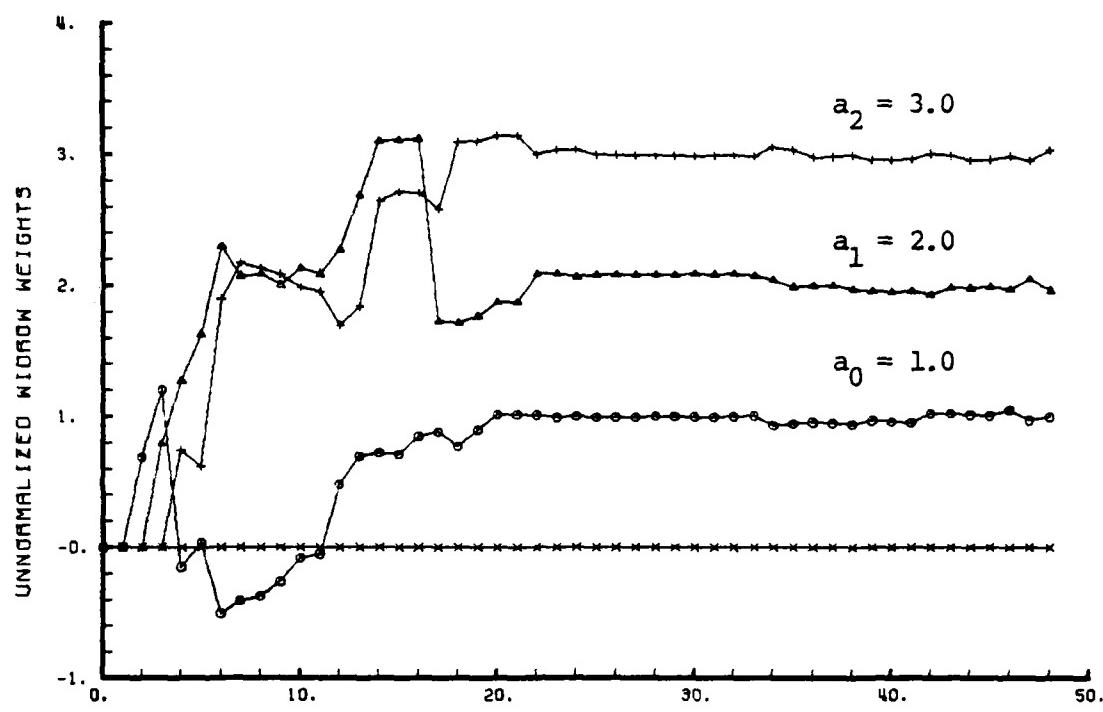


Figure 8.5b. LMS Adaptive Filter Weights, $k_s = -.200$, with increased measurement noise

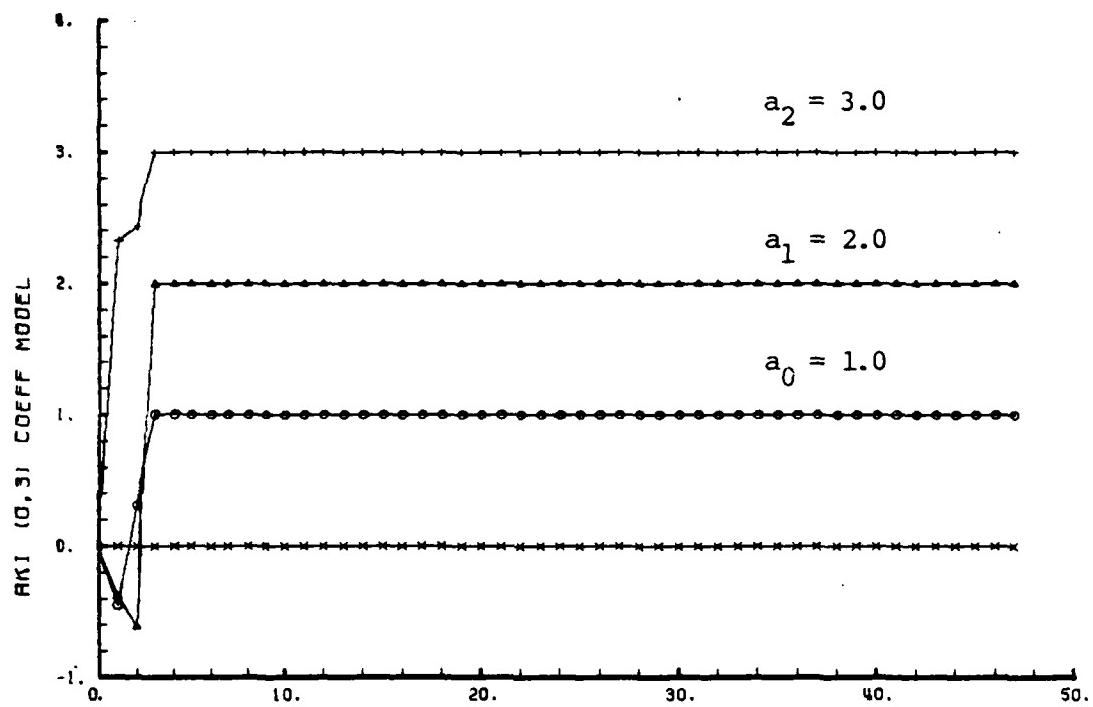


Figure 8.6. AKI Moving Average Plant Coefficients

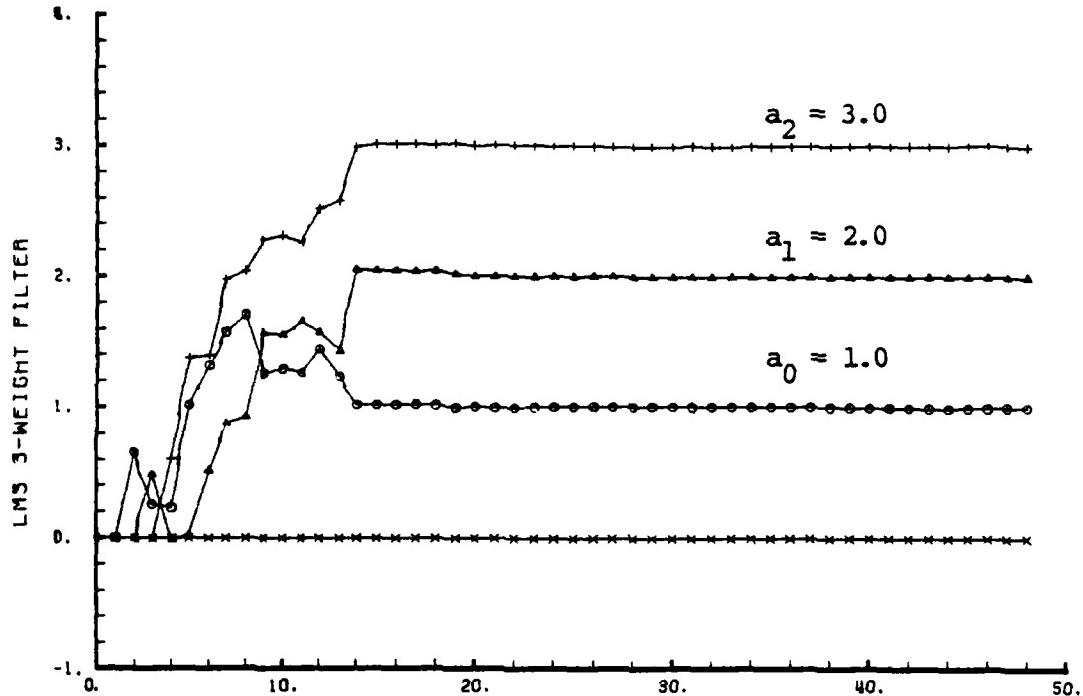


Figure 8.7. LMS Adaptive Filter Weights

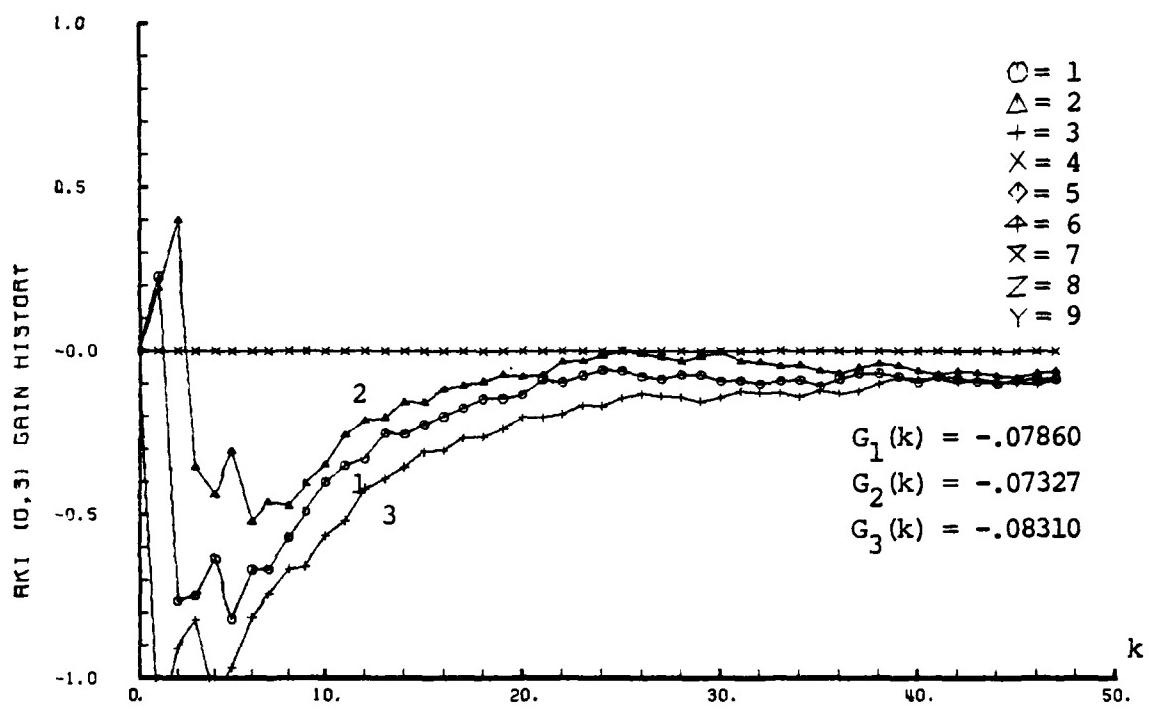


Figure 8.8. AKI Averaged Gains for Moving Average Model

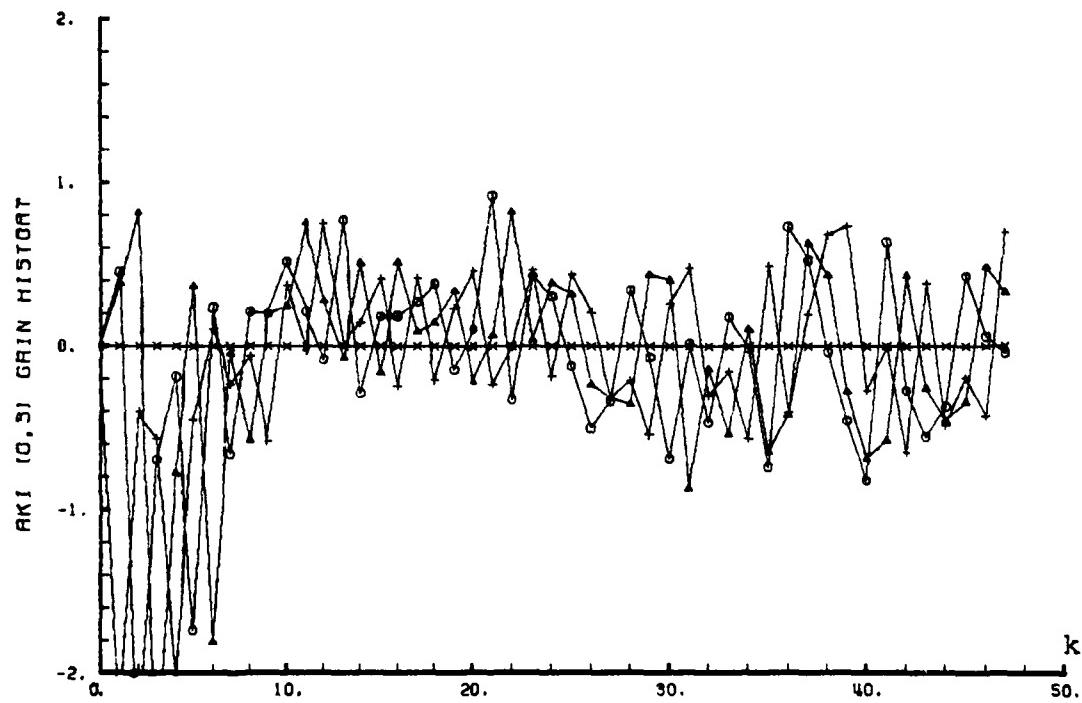


Figure 8.9. AKI Moving Average Instantaneous Gains

$E\{\underline{G}(k)\}$ is the present average value for the gain

$\underline{G}(k)$ is the present value of the instantaneous gains

$\frac{1}{k+1}$ is the averaging gain.

This method of obtaining a "running" average is generally well known, (see for example Ref. 39). The example presented was not the only one used but a good representative of the operation of the AKI when the data was derived from a moving average process.

2. AKI Applied to Autoregressive Data

Before applying the AKI to identifying the coefficients of a complex ARMA plant, it was first tested using data derived from a plant whose transfer function specified an autoregressive process. The transfer function used was,

$$\begin{aligned} H(z) &= \frac{1}{1 + \frac{26.0}{24.0}z^{-1} + \frac{9.0}{24.0}z^{-2} + \frac{1.0}{24.0}z^{-3}} \\ &= \frac{a_0}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3}} \end{aligned} \quad (8.3)$$

The AKI had no problem converging on the plant coefficients (Figure 8.10) producing the following estimates at the 42nd iteration (Table 8.1). It essentially converged on the plant coefficients in approximately five iterations. The gains computed by the AKI for this case also displayed convergence to a steady state value. The gain history for this example is depicted in Figure 8.11.

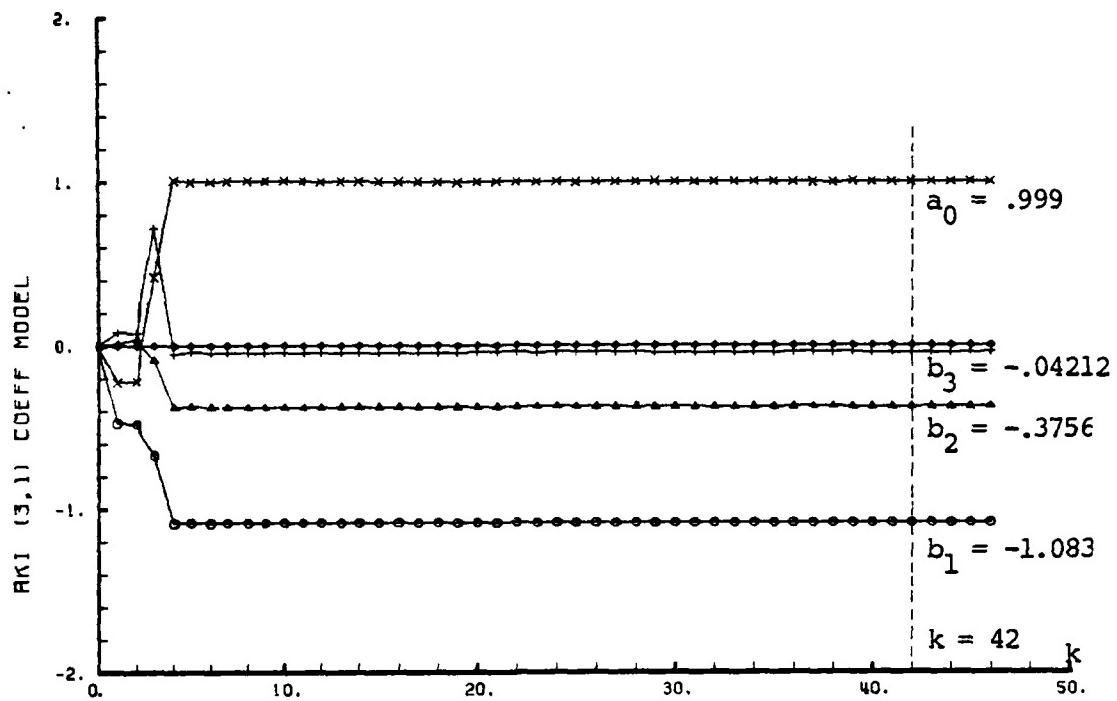


Figure 8.10. AKI Autoregressive Plant Coefficients

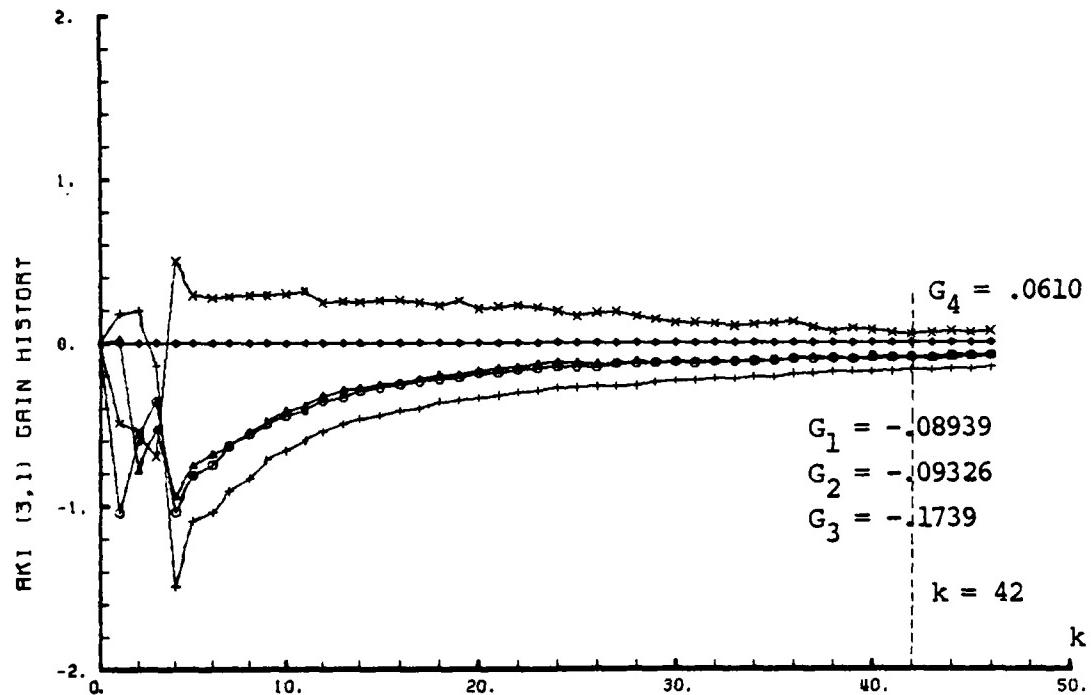


Figure 8.11. AKI Autoregressive Averaged Gains

Table 8.1
ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION 8.3

	ACTUAL	AKI ESTIMATES
a_0	1.0	0.9995
b_1	-1.08333 ...	-1.083
b_2	-0.3750	-0.3756
b_3	-0.04166 ...	-0.04212

3. AKI Applied to ARMA Data

The next logical step was to use the AKI to identify the coefficients of a general ARMA process. One of Perry's models [Ref. 6] was used for this purpose. Specifically, the transfer function of the plant was,

$$\begin{aligned} H(z) &= \frac{1.0 + 1.4z^{-1} + .98z^2}{1 - 1.14z^{-1} + 1.4549z^{-2} - .88490z^{-3} + .40745z^{-4}} \\ &= \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{1 - b_1 z^{-1} - b_2 z^{-2} - b_3 z^{-3} - b_4 z^{-4}} \end{aligned} \quad (8.4)$$

The ARMA plant was subjected to the same conditions which Perry [Ref. 6] describes. That is, unit variance, zero mean, white gaussian noise was used as the input. The reader is reminded that the output signal processed by the AKI was corrupted by measurement noise as described in Chapter V. The output data used in Perry's examples, however, reflects noiseless measurements. Table 8.2 tabulates the results for the coefficient estimates computed by the AKI. Even though the results presented in Table 8.2 represent the coefficient

Table 8.2

ACTUAL VS AKI ESTIMATES FOR COEFFICIENTS OF EQUATION
8.4 AT THE 371ST ITERATION

	ACTUAL	AKI ESTIMATES
a_0	1.00000	0.98760
a_1	1.40000	1.40100
a_2	0.98000	0.99280
a_3	0.00000	0.01252
a_4	0.00000	-0.00922
b_1	1.14000	1.14000
b_2	-1.45490	-1.45900
b_3	0.88490	.88740
b_4	-0.40745	-.40980

estimates at the 371st iteration, it can be seen from Figure 8.12 that the AKI has essentially converged by the 28th iteration. We note also the characteristic convergence of the averaged gains to steady state values in Figure 8.13. As a means of comparison with Perry's results, the poles and zeros of the AKI estimates at the 28th, 90th and 371st iteration are plotted in Figure 8.14a and Figure 8.14b. Perry's results Figure 3.7 [Ref. 6:pp. 107,108] for his lattice modeling of the plant represented by equation (8.4) are reproduced for convenience.

As was noted for the previous cases, the instantaneous gains appeared erratic, Figure 8.15, whereas the averaged gains converged to some steady state value.

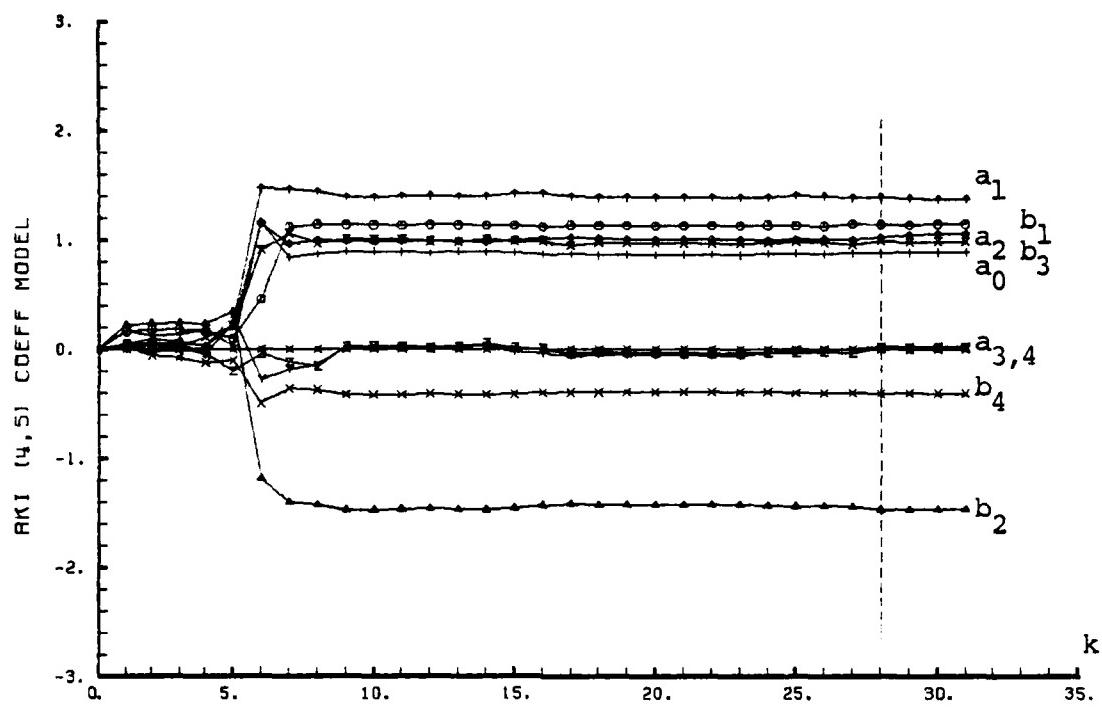


Figure 8.12. AKI(4,5) Computer Coefficients, Perry's Example

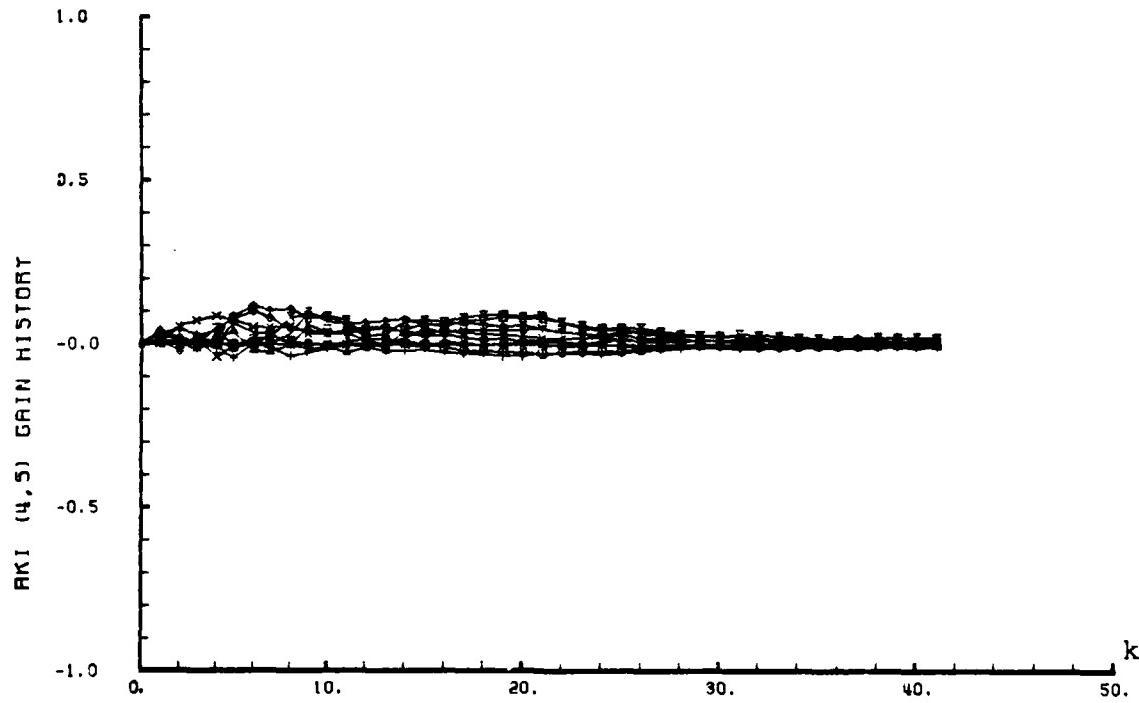


Figure 8.13. AKI(4,5) Averaged Gain Values

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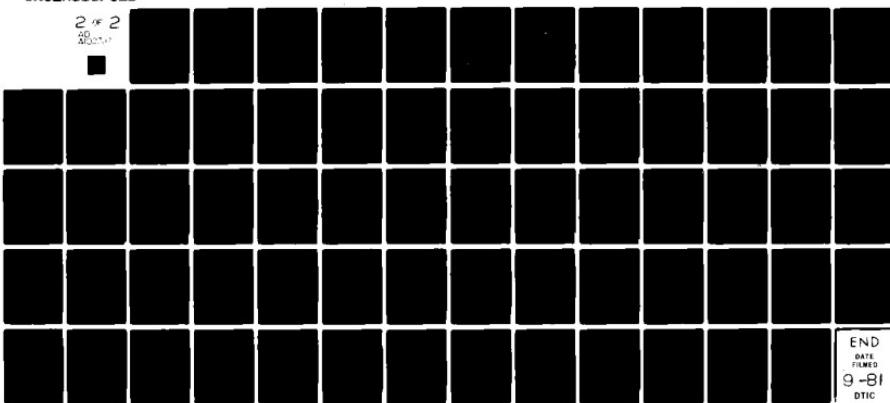
NAVAL POSTGRADUATE SCHOOL MONTEREY CA
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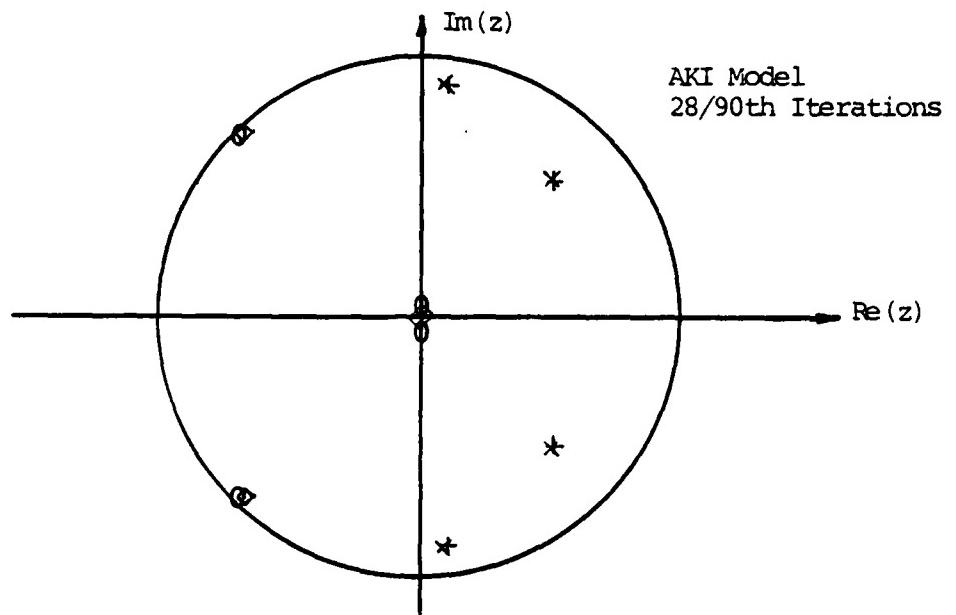
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END
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X Model Pole
0 Model Zero
+ System Pole
◊ System Zero

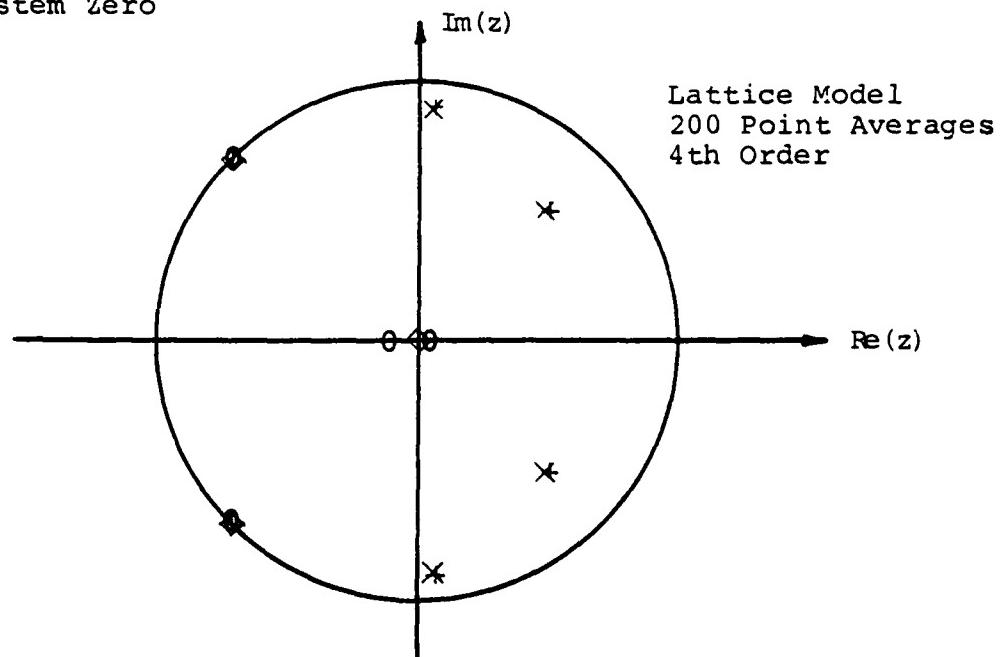
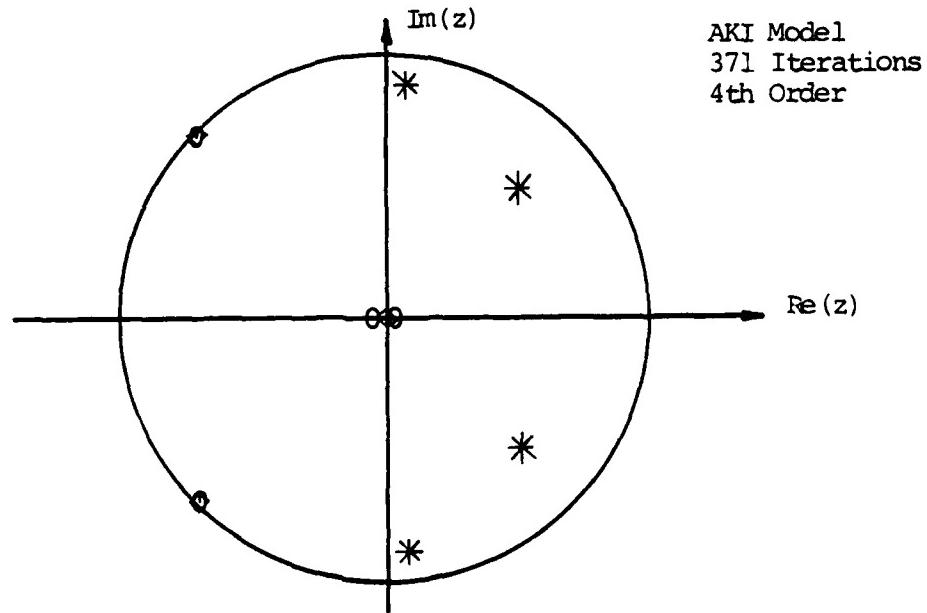


Figure 8.14a. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4)



X Model Pole
0 Model Zero
+ System Pole
◊ System Zero

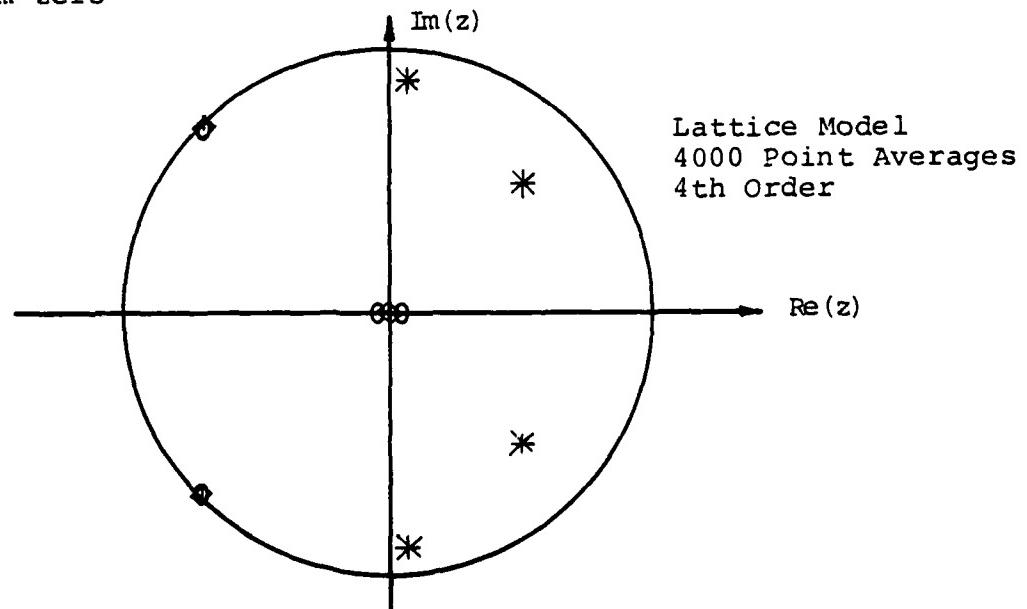


Figure 8.14b. Pole/Zero Models Produced by AKI and Lattice Algorithms for a Plant with the Characteristic Transfer Function of Eqn. (8.4) at 371st Iteration

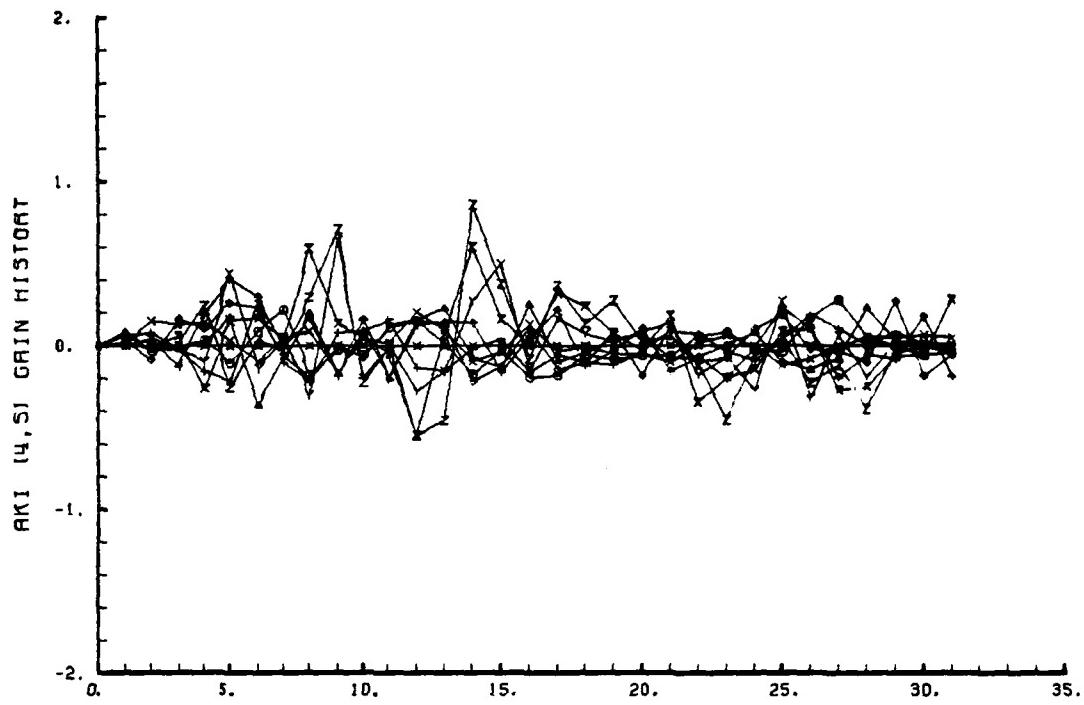


Figure 8.15. Instantaneous Gain Values, AKI(4,5) Model

4. AKI vs Adaptive Recursive LMS Filter

Using Feintuch's proposed algorithm [Ref. 18] and repeating the simulation presented in the rebuttal to Johnson and Larimore [Ref. 19], a comparison was made between the operation of the AKI and the Adaptive Recursive LMS filter. The Adaptive Recursive LMS filter required "tuning" of the convergence factors, k_a and k_b , to what appeared to be the optimal performance features of (1) fast convergence and

(2) stable estimates. The "tuning" process was a trial and error procedure beginning with the convergence constants given by Feintuch,

$$k_a = -4.3 \times 10^{-4} \quad (8.5a)$$

$$k_b = -4.3 \times 10^{-4}. \quad (8.5b)$$

The Adaptive Recursive LMS algorithm was then implemented to estimate the coefficients of the plant whose transfer function was,

$$H(z) = \frac{0.05 - 0.40z^{-1}}{1 - 1.1314z^{-1} + 0.25z^{-2}} \quad (8.6a)$$

$$= \frac{a_0 + a_1 z^{-1}}{1 - b_1 z^{-1} - b_2 z^{-2}} \quad (8.6b)$$

The best response obtained by trial and error resulted in the convergence constants,

$$k_a = -4.3 \times 10^{-3} \quad (8.7a)$$

$$k_b = -14.3 \times 10^{-3}. \quad (8.7b)$$

Feintuch reported that after $8,192^2$ iterations the estimates had converged on the coefficients of (8.6a) with a .2096 normalized rms error. Using the convergence constants (8.7) convergence was essentially reached by the 4,000th iteration.

²Feintuch reports the resulting estimates for the coefficients at several intermediate iterations from 8,192 to 65,536, however 8,192 was the minimum number of iterations reported.

It was felt that the operation of the adequately tuned Adaptive Recursive LMS filter could be fairly compared with the operation of the AKI.

Table 8.3 tabulates the performance of the two system identification methods and Figures 8.16 and 8.17 graphically present the responses. Figure 8.18 not only shows at what point the gains of the AKI reach a constant value but also

Table 8.3

ACTUAL VS AKI AND ADAPTIVE RECURSIVE LMS FILTER ESTIMATES FOR COEFFICIENTS OF EQUATION 8.6

	ACTUAL	AKI ESTIMATES (k = 150)	ADAPTIVE RECURSIVE LMS (k = 3,990)
a ₀	0.05	0.05107	0.05045
a ₁	-0.40	-0.4007	-0.3990
b ₁	1.1314	1.13200	1.12800
b ₂	-0.25	-0.25090	-0.2442

provides a measure of confidence that the unknown system has been identified. It is evident from Figure 8.19 that both methods seem to identify the poles and zeros of the actual plant; however, the poles computed by the AKI are closer. The zeros computed by the AKI are approximately .080 units farther from the true zero than is the Adaptive Recursive LMS filter estimate. All aspects considered, the AKI takes considerably fewer iterations to arrive at its estimate.

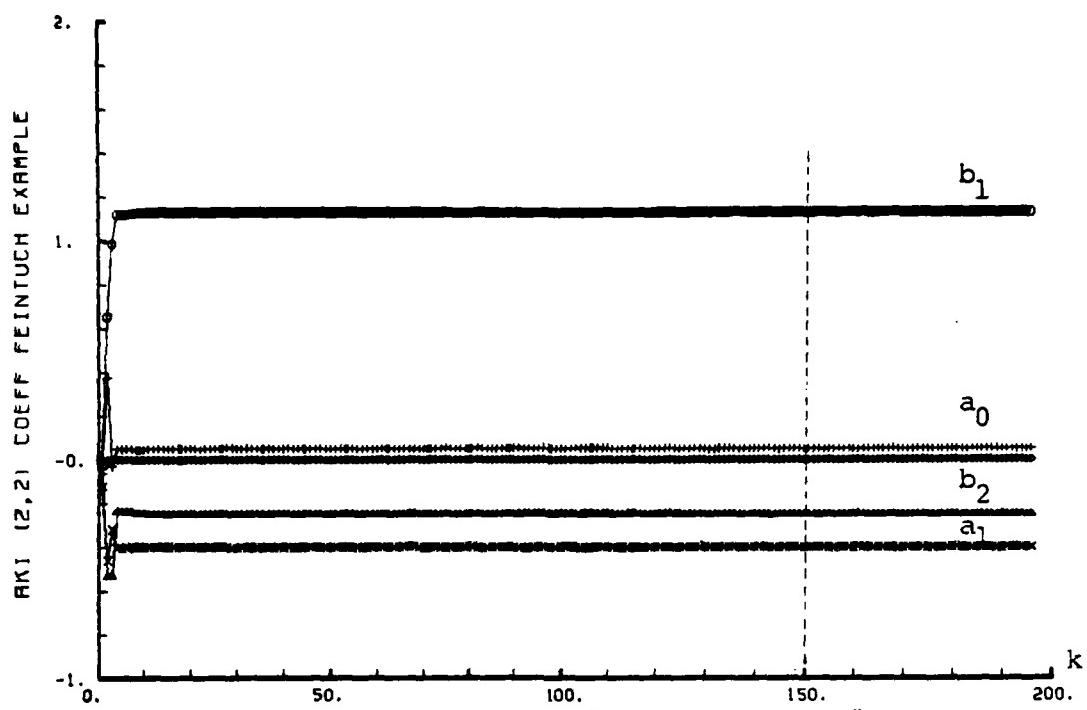


Figure 8.16. AKI(2,2) Computed Coefficients, Feintuch Example

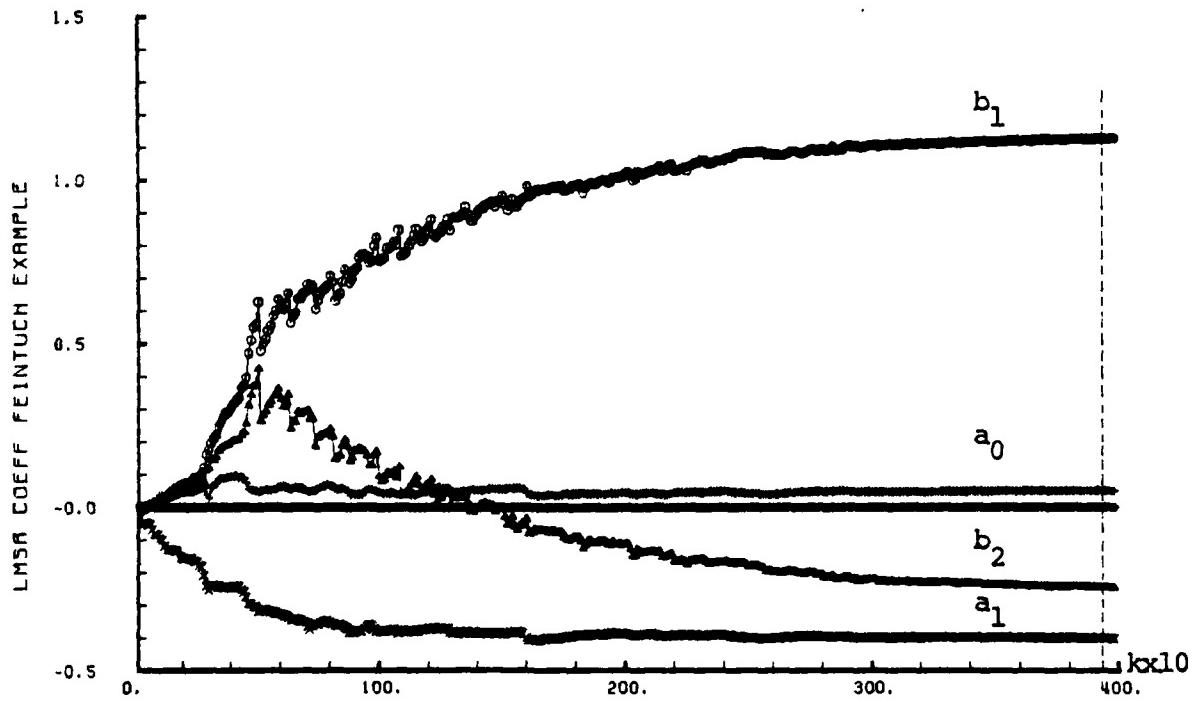


Figure 8.17. LMSR Computed Coefficients, Feintuch Example

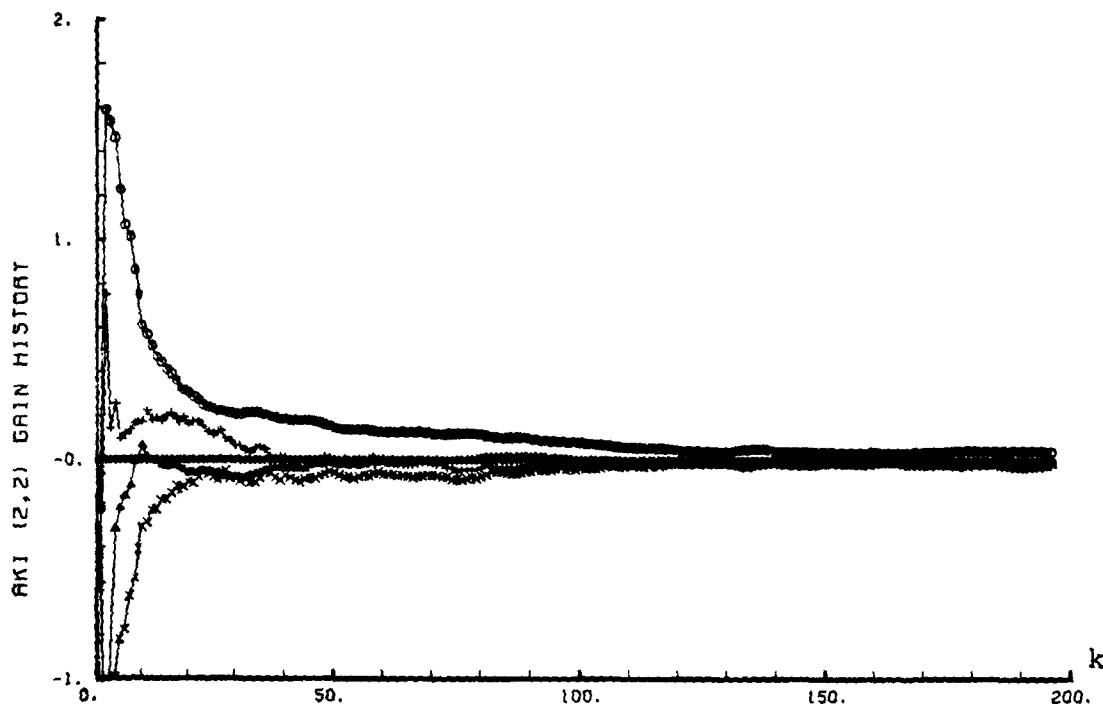


Figure 8.18. AKI(2,2) Averaged Gains, Feintuch Example

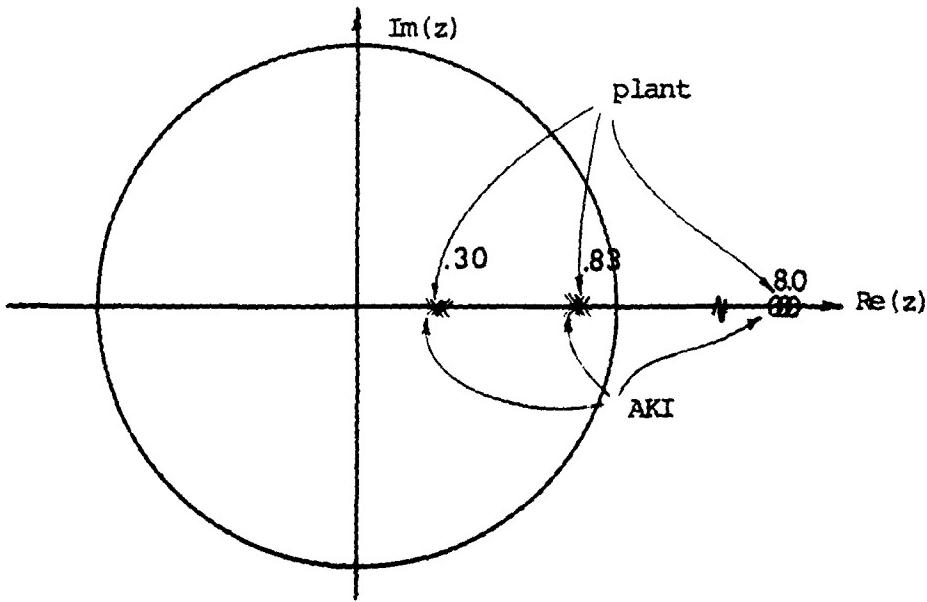


Figure 8.19. Pole/Zero Models Produced by AKI and Adapt.
Rec. LMS Algorithms for a Plant with the
Characteristic Transfer Function of Eqn. (8.6)

5. AKI Applied to PLL Data (Linear Region)

To identify the coefficients associated with the general autoregressive-moving average representation for the phase locked loop, the input noise signal power (mean square value) was kept at $(10 \text{ deg})^2$. In this manner, the linear region of the PLL was invoked. The input/output data was analyzed by the AKI resulting in the estimates shown in Table 8.4. Figures 8.20 and 8.21 show the response of the AKI

Table 8.4

ACTUAL VS AKI ESTIMATES OF THE ARMA REPRESENTATION FOR THE PLL (LINEAR REGION)

	ACTUAL	AKI ESTIMATES ($k = 82$)	AKI ESTIMATES ($k = 350$)
a_0	0.000000	-0.0001046	-0.000692
a_1	0.020000	0.0198500	0.0199800
a_2	-0.018890	-0.018720	-0.0188900
b_1	1.980000	1.980000	1.978000
b_2	-0.981110	-0.981000	-0.9790

when applied to the identification of the PLL data. It was noted that though the AKI correctly identified the coefficients of the linear PLL, the AKI gains were large due to the weak signals (input/output data) incorporated in the measurement vector, $H(k)$.

6. AKI Applied to PLL Data (Non-linear Region)

When analyzing any non-linear system the engineer must bring to bear all his analysis techniques on the problem. The PLL was therefore studied using classical root locus

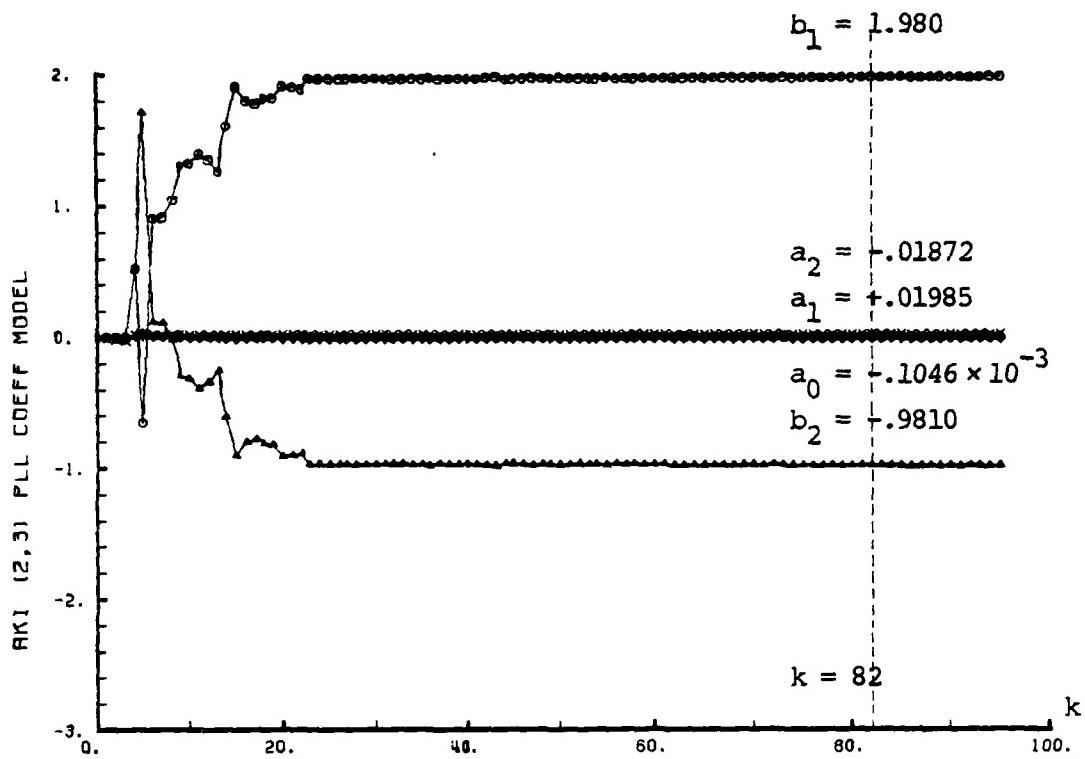


Figure 8.20. PLL, AKI(2,3) Computed Coefficients

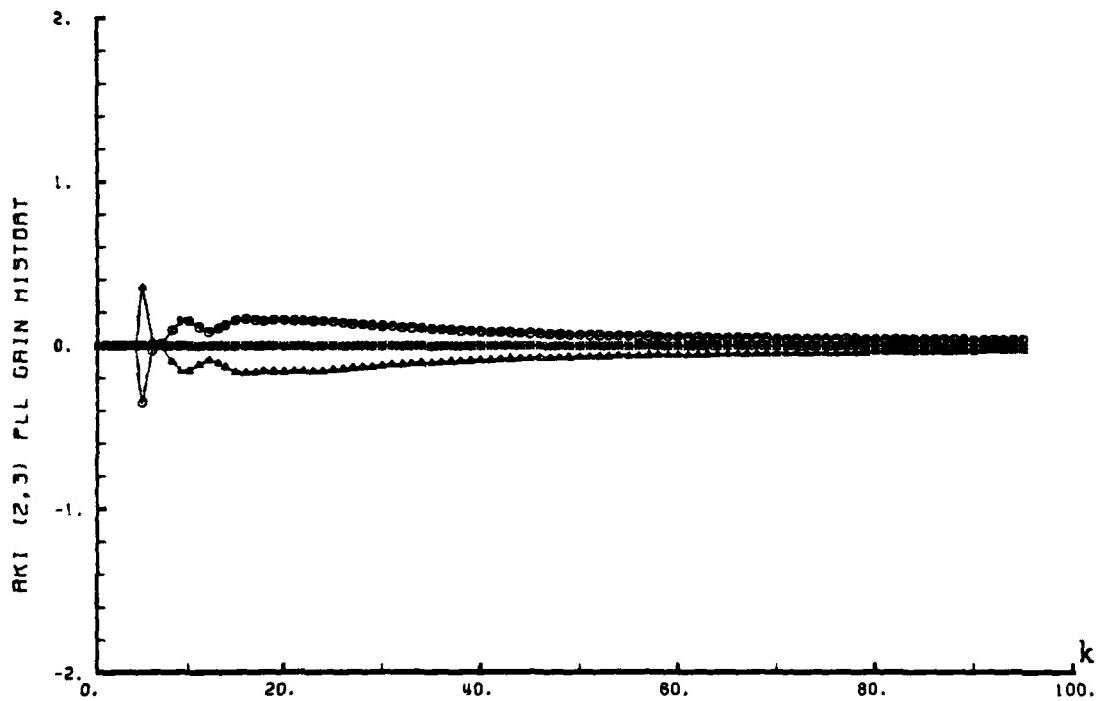


Figure 8.21. Averaged Gain Values, AKI(2,3) PLL Model

techniques to preview the possible outcomes when being modeled by the AKI. Stability analysis of a more complex PLL system using the root locus technique has been previously presented in the literature [Ref. 40].

The root locus technique can be applied to the PLL presented in this thesis by first assuming that the $\sin(\cdot)$ block of Figure 5.7 is a variable gain, λ . This is not a restrictive assumption since the PLL during operation generally tracks small deviations. The loop gain can be written by inspection as,

$$\begin{aligned} L(z) &= \frac{\lambda(0.02)(1 - 0.9445z^{-1})z^{-1}}{(1 - z^{-1})^2} \\ &= \frac{\lambda(0.02)(z - 0.9445)z}{(z - 1)^2} \end{aligned} \quad (8.8)$$

From equation (8.8) the root locus of $L(z)$ is drawn as shown in Figure 8.22. Even though the root locus technique is generally used when the signals in the system are considered deterministic, it is not surprising that some of the results obtained are nevertheless valid. When a moderately strong input noise identification signal [$E\{u^2(u)\} < (25 \text{ deg})^2$] was used, the pole-zero locations of the PLL seem to follow the classical root locus analysis. However, when the input noise identification signal power is increased beyond $(25 \text{ deg})^2$ the pole-zero locations do not follow the expected behavior predicted using the root locus method as can be seen in Figure 8.22.

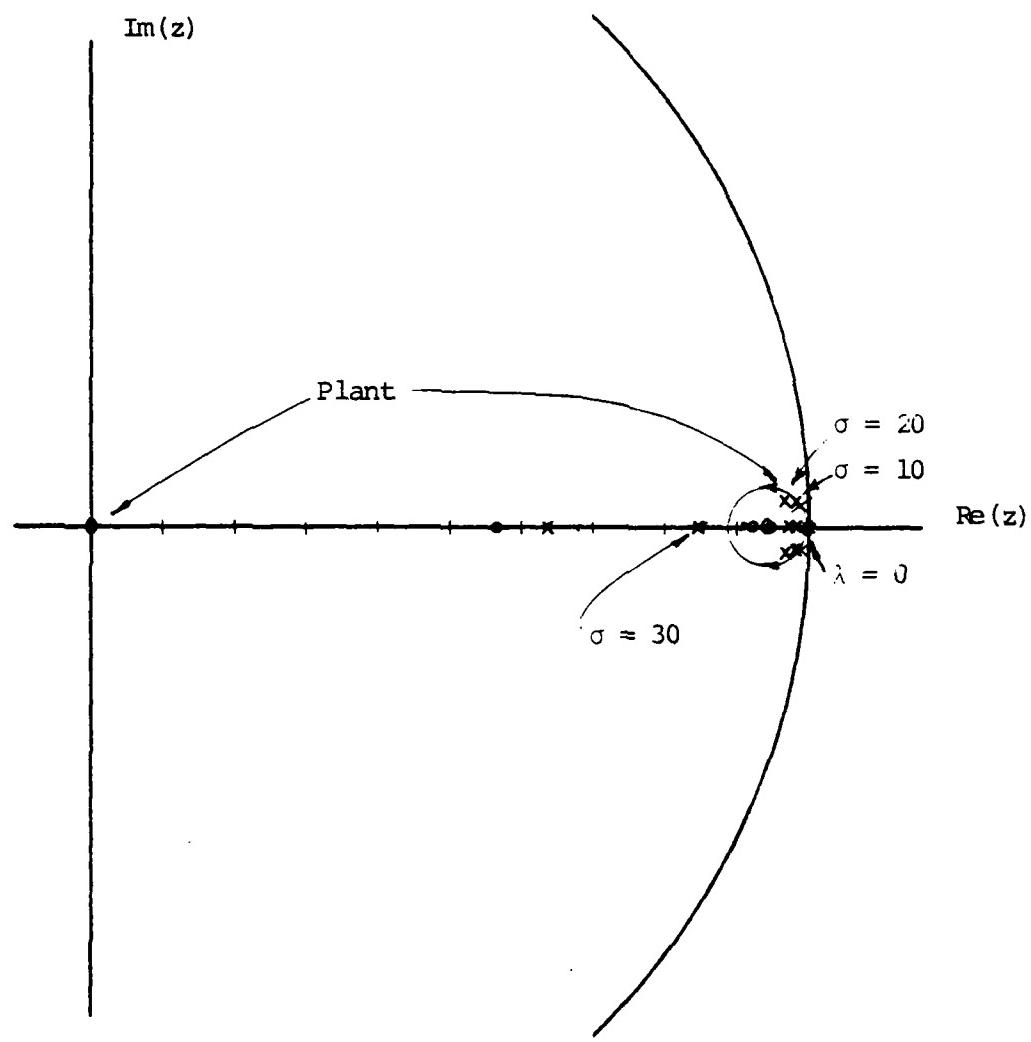


Figure 8.22. PLL Root Locus Analysis vs Computer Roots
of the AKI ARMA Model

The departure of the pole-zero behavior from what was expected was analyzed by closer investigation of the linear terms of equation (7.8).

$$\begin{aligned} y(k) &= .02\alpha u(k-1) - .01889\alpha u(k-2) + (2.0 - .02\alpha)y(k-1) \\ \text{lin} &\quad + (.01889\alpha - 1)y(k-2) \end{aligned} \quad (8.9)$$

Recalling that a third order Taylor series approximation of the sine is the functional expressed by equation (7.7),

$$\sin(x) \approx \alpha x + \beta x^3 \quad (7.7)$$

one notes that the linear region is described when $\alpha = 1$ and $\beta = 0$. It is therefore reasonable to study the variation of α with respect to input noise power.

The average value $\bar{\alpha}$ of α , was computed by equating the estimated AKI coefficients to the coefficients of like terms in equation (8.9) for several input noise power levels. The relation used was

$$\bar{\alpha} = \frac{1}{4} \left[\frac{2.0 - \hat{b}_1}{.02} + \frac{1.0 + \hat{b}_2}{.01889} + \frac{\hat{a}_1}{.02} - \frac{\hat{a}_2}{.01889} \right] \quad (8.10)$$

at the 350th iteration. The relationship between $\bar{\alpha}$ and the input noise power level is readily apparent from Figure 8.23. This result is plausible since if one considers the input-output relationship of the sine block of Figure 5.7 one obtains Figure 8.24. Superimposing the gaussian probability functions of the different input noise signals, it can be seen that for

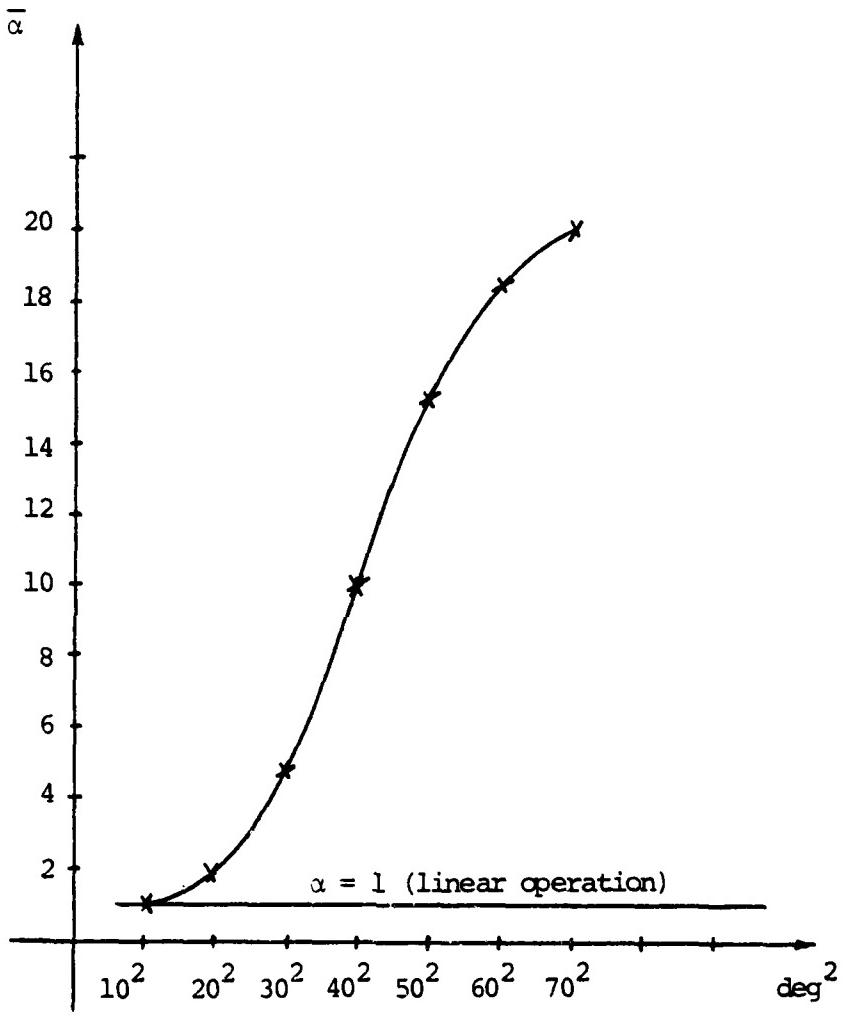


Figure 8.23. Variation of $\bar{\alpha}$ vs Input Noise Signal Power

low power levels of the input signal, identification of the linear parameters of the overall system (equation 5.9) can be made. As the input power is increased beyond (25 deg)² the input-output characteristics of the sin x block are no longer approximately linear, showing its effect in Figure 8.24 as a departure from its linear operation, $\bar{\alpha} = 1$. Therefore, by monitoring $\bar{\alpha}$ one can determine when the overall system is entering its non-linear operating regime.

Since the same functional dependence between each of the AKI estimates and $\bar{\alpha}$ did not exist for all of the input noise powers considered, knowing $\bar{\alpha}$ did not provide any information of the pole locations but did provide a measure of the degree of non-linear operation.

B. ORDER OF THE UNKNOWN SYSTEM IS NOT KNOWN (OVERMODELING)

1. AKI vs LMS Adaptive Filter

Using the data derived by operating a plant with the transfer characteristic of equation (8.1) the operation of the AKI and the LMS adaptive filter was compared when the orders used in the identifier and the LMS filter were greater than the known process that generated it. The following overmodeling cases were studied:

- (1) MA model is greater than plant MA process
- (2) ARMA model is fitted to MA plant

When the MA model order is greater than that of the plant MA process, it was found that both the AKI and the LMS filter would compute coefficients close to zero for the higher

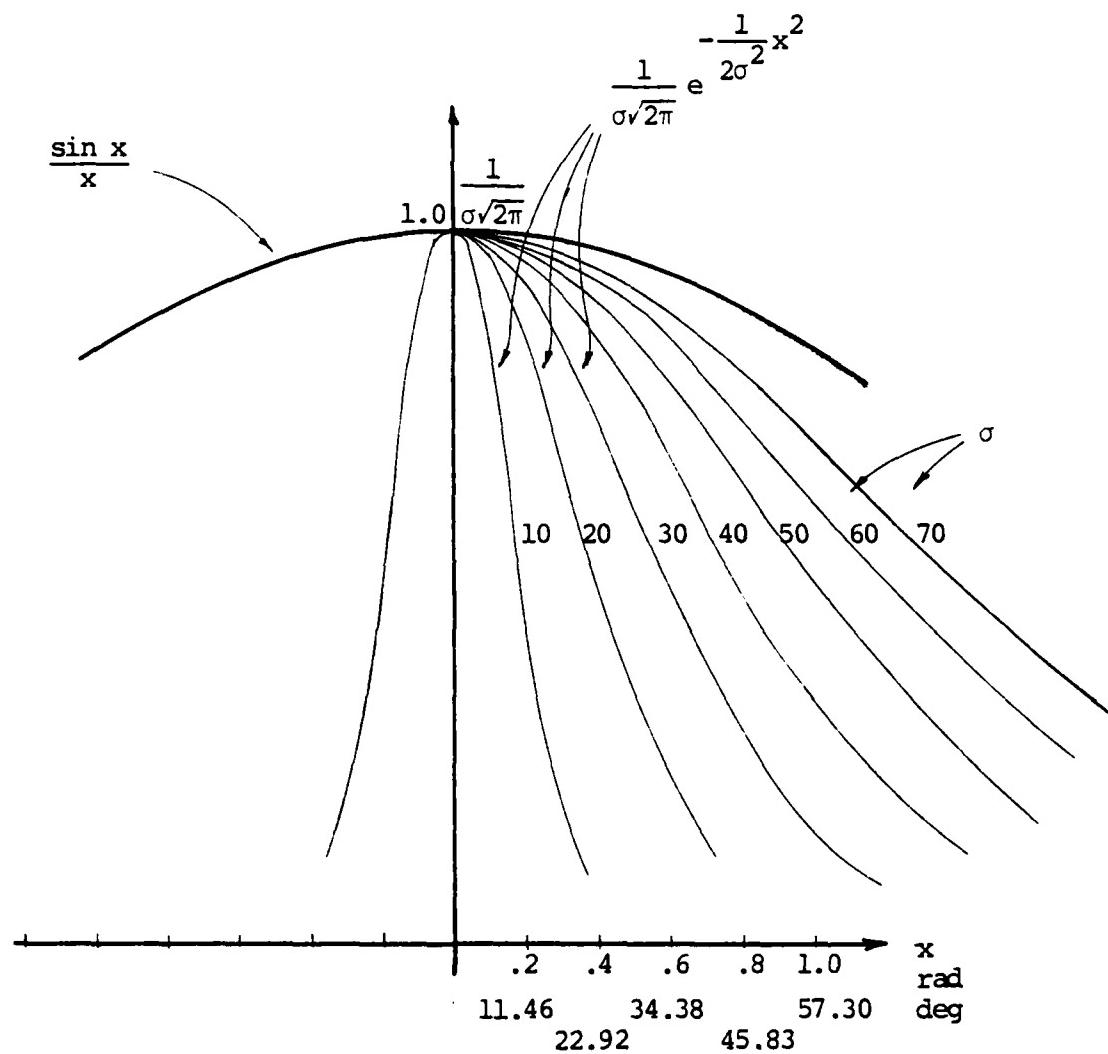


Figure 8.24. Comparison of $\frac{\sin x}{x}$ and p.d.f. of input noise signals

order coefficients. This effect can be seen in Figures 8.25a and 8.25b when a fourth order MA process is used to model the actual second order process represented by equation (8.1). Since these are a "zeros only" plant and model the overmodeling essentially causes zeros to appear at the origin of the Z-plane of the model transfer function. Table 8.5 summarizes the results for two overmodeling conditions at the 250th iteration using the AKI. When a purely moving average process represented by equation (8.1) was modeled as an ARMA process, one must direct his attention to the poles and zeros of the model transfer function which the AKI computed and compare them to the actual plant poles and zeros. It was not readily apparent from the resulting coefficients that the plant had been identified. The following example will help clarify what is happening.

For the data produced by the second order plant (equation 8.1), an autoregressive moving average (ARMA) model of orders 2 and 5 respectively was fitted. It can be seen from Table 8.6 that no firm conclusions can be drawn about which coefficients actually identify the plant. We, the analysts, knowing the form of the plant which produced the data, could qualitatively state that b_1 , b_2 , a_3 , a_4 and a_5 are small enough to be ignored. Hence, we can identify the plant correctly. However, inspection of the poles and zeros of the transfer function which the AKI computed,

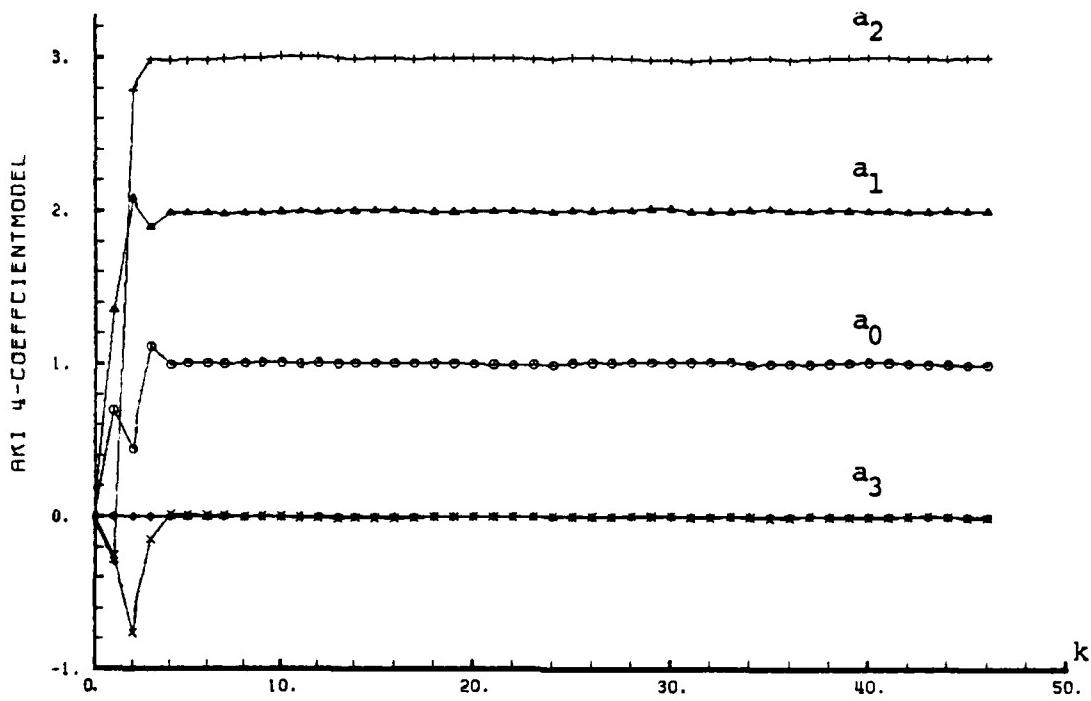


Figure 8.25a. AKI(0,4) Model of a Third Order MA Process

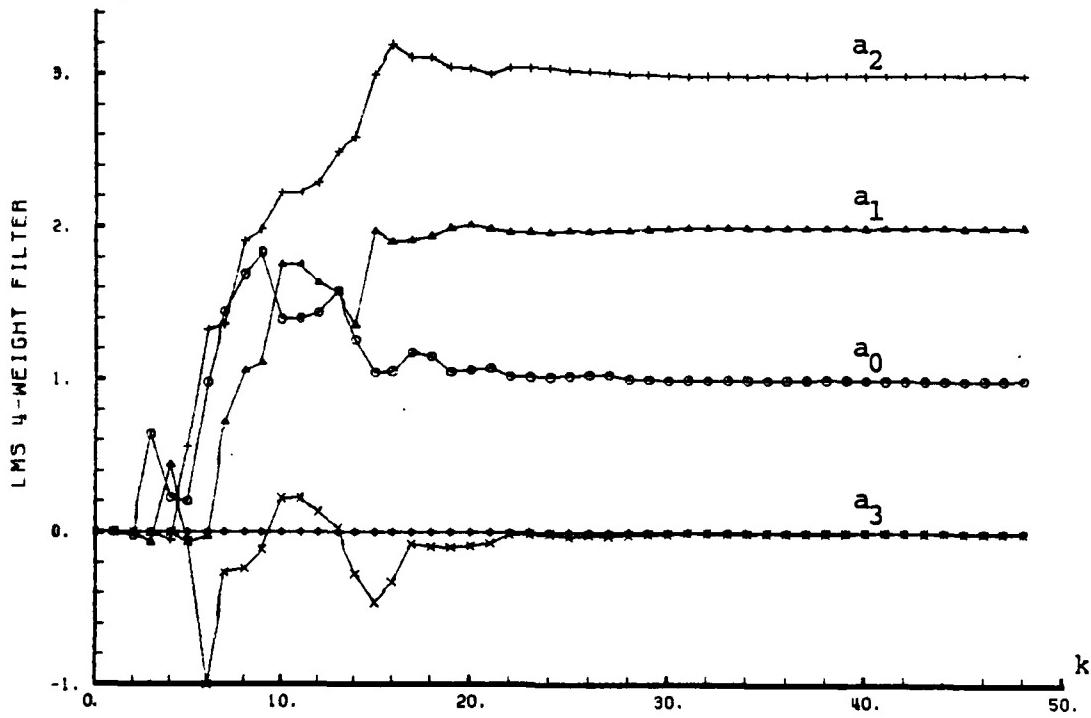


Figure 8.25b. Fourth Order LMS Filter Weights of a Third Order MA Process

Table 8.5
OVERMODELING OF A MA PROCESS USING THE AKI ($k = 250$)

	Actual Model Coeffs	3rd Order AKI MA Model	5th Order AKI MA Model
a_0	1.0	.9995	.9991
a_1	2.0	2.0000	2.000
a_2	3.0	3.0000	3.000
a_3	0.0	$.893 \times 10^{-3}$.0003191
a_4	0.0	---	.001082
a_5	0.0	---	.0003198
zeros:	$-1.0 \pm 1.414j$	$-1.000 \pm 1.414j$ $-.2977 \times 10^{-3}$	$-1.001 \pm 1.414j$.0227 $\pm .0428j$ -.4534

Table 8.6
OVERMODELING OF A MA PROCESS USING THE AKI OF ORDERS
AR = 2, MA = 6

	Actual Plant Coefficients	AKI(2,6) Model Coefficients
b_1	0.0	.00467
b_2	0.0	.10500
a_0	1.0	1.0000
a_1	2.0	1.9940
a_2	3.0	2.8840
a_3	0.0	-0.2257
a_4	0.0	-0.3142
a_5	0.0	0.0001517

$$H_{AKI(2,6)}(z) = \frac{1.0 + 1.994z^{-1} + 2.8840z^{-2} - 0.2257z^{-3} - 0.3142z^{-4} + 0.001517z^{-5}}{1 - .00467z^{-1} - .1050z^{-2}}$$

(8.11)

results in:

poles:	-.8217
	.3264
zeros:	-.9996 ± 1.414j
	.4827 × 10 ⁻³
	-.3212
	.3262

The observation to be made is that there are two pole-zero combinations which are near cancellation. This suggests that the AKI algorithm be rerun with the AR and MA orders reduced by at least two.

The implication of the analysis of this section is that a model can in theory be found for a given set of input/output data. Further, a parsimonious model can be identified by careful observation of pole-zero combinations which are near cancellation and of stray zeros near the origin.

2. AKI Applied to AR and ARMA Data

Essentially the same characteristic results found in Section VIII.B.1 were confirmed when the data produced by plants defined by the transfer functions of equations (8.3) and (8.4) were analyzed by the overmodeled AKI. That is, pole-zero pairs near cancellation and zeros near the origin

were produced by the AKI. Figures 8.26 through 8.28 have been chosen as representative pole-zero plots of the transfer functions computed by the AKI for the AR plant of Section VIII.A.2 and for Perry's model (Section VIII.A.3).

C. CONCLUSION

This work indicates that the Kalman filter algorithm heretofore generally used as a state estimator, or in augmented form to estimate parameters (in which case the parameters are treated as states), can also be formulated in an adaptive manner to iteratively estimate the coefficients of an ARMA equation explicitly. This approach, termed the Adaptive Kalman Identifier (AKI), summarily identifies the unknown system whose input/output data is being processed. The LMS adaptive algorithm of Widrow, and its modification by Griffiths (in which the convergence factor is selected to be inversely proportional to the input signal power) are shown to be suboptimal cases of the AKI. An additional insight provided by the AKI is that it indicates clearly how measurement noise might be taken into account in the LMS adaptive formulation.

The operation of the AKI was checked by way of simulation and compared with two existing identification techniques:

(1) the LMS Adaptive nonrecursive (MA) algorithm and (2) the Adaptive Recursive ARMA LMS algorithm. It was found that not only are the two LMS filtering techniques special suboptimal cases of the AKI; but, further, the AKI exhibits superior convergence and modeling properties for the cases where (1) the

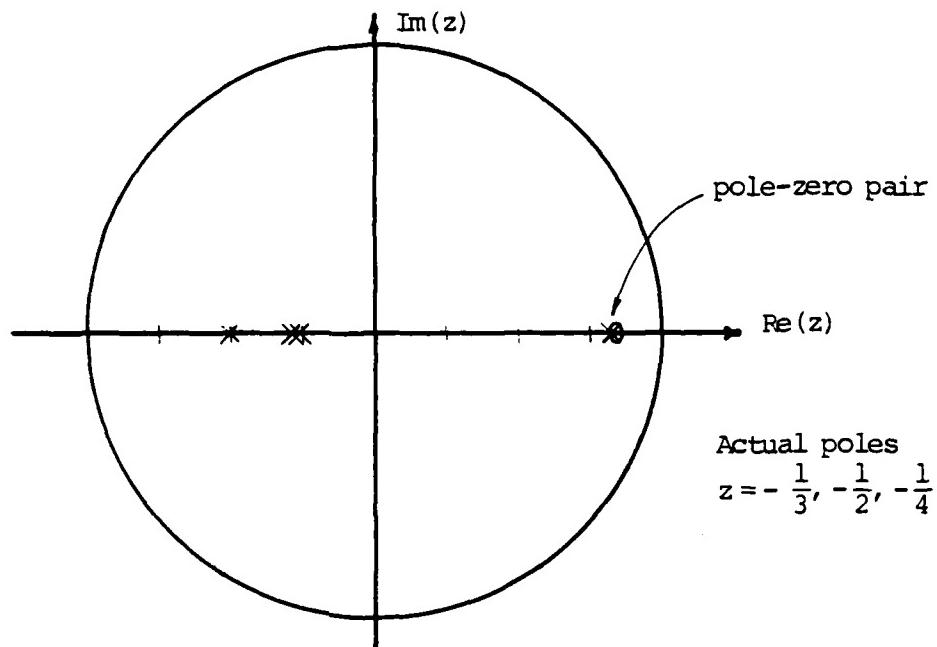


Figure 8.26. AKI(3,1) Overmodeled AKI(4,2)

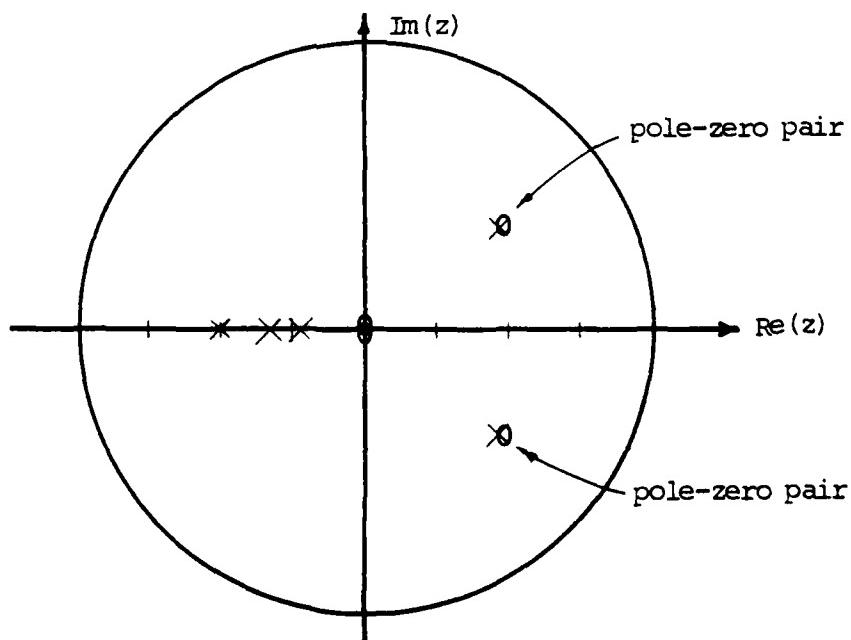


Figure 8.27. AKI(3,1) Overmodeled AKI(5,5)

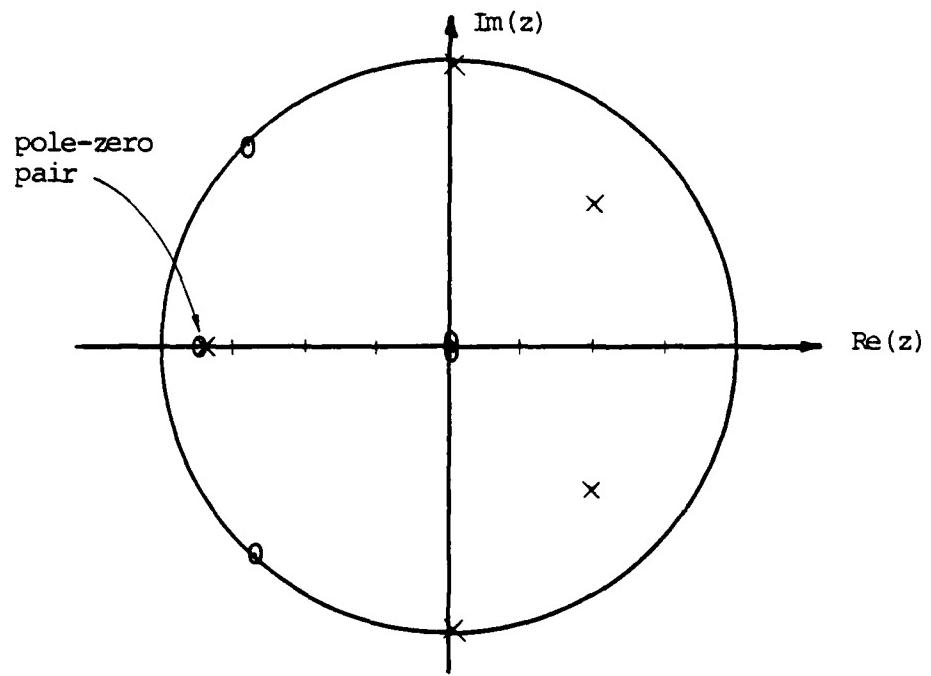


Figure 8.28. AKI(4,5) Overmodeled AKI(5,6)

order of the unknown plant is known and (2) the order of the plant is overmodeled. Additionally, the simulations indicate that accuracies similar to those obtained using lattice modeling methods, can be achieved using the AKI at a decidedly smaller number of iterations.

By making minor modifications to the measurement vector, $H(k)$, that is by using hybrid signals, the AKI was used to identify the linear and non-linear ARMA representations of a phase locked loop with success. Interestingly, the AKI technique appears to enable one to discern when a potential non-linear system enters its non-linear mode of operation, by closely monitoring the coefficients of the linear portion of the generalized non-linear ARMA model.

D. TOPICS FOR FURTHER CONSIDERATION

Several areas for further study directly and indirectly related to the AKI were uncovered. Foremost, a rigorous convergence proof is desirable. Although the connection was made between the AKI algorithm and the initial equation from which the lattice modeling algorithm is developed, similar comparisons (such as Chapters III and IV) could provide more insight into the operation of both. The multichannel AKI is the logical development of the single channel AKI presented. And, lastly, refinement of the AKI software (using the NL = 5 option) to include ARMA modeling of time series using a Box-Jenkins approach [Ref. 26] is feasible. A limited number of simulations using Monterey rain data and series C [Ref. 26]

data seem to indicate that an application exists for the AKI
in this area.

APPENDIX A
THE DISCRETE WEINER PROBLEM

The discrete Weiner problem considers optimally filtering a desired signal from unwanted stationary noise. The criterion of optimality used is minimization of the mean squared error between the output and the desired signal. Generally one desires that the optimal filter, which is the device being sought, be time invariant and that it be able to accept a signal, $s(k)$, and noise, $n(k)$, where each are samples from stationary random processes. In other words, we want a device which can accept

$$x(k) = s(k) + n(k) \quad (A.1)$$

as an input and produce at its output $s(k+\Delta)$ or some linear function thereof where Δ is some known delay.

Assuming that the desired output, $d(k)$, is the response to a specified sampled data linear system whose transfer function, $H_d(z)$, is given, then the error between the desired output and the output of the filter, $y(k)$, we seek is,

$$e(k) = d(k) - y(k). \quad (A.2)$$

Figure A.1 depicts the discrete Weiner problem formulation. The derivation from here follows the one presented by Maybeck [Ref. 27] for the continuous case. From the block diagram we have,

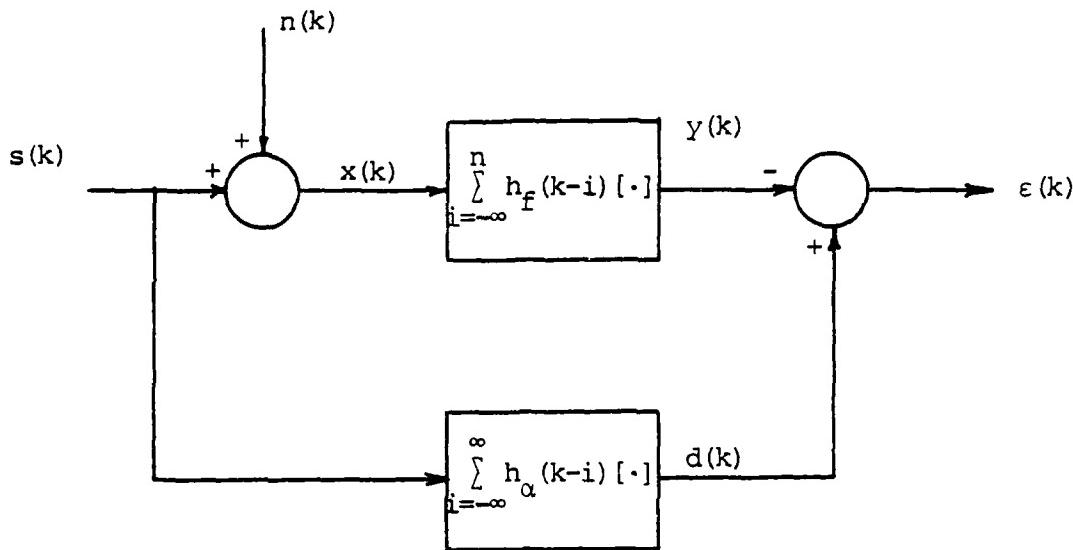


Figure A.1. The Discrete Weiner Problem

$$y(k) = \sum_{i=-\infty}^{\infty} h_f(k-i) x(i) \quad (A.3)$$

or equivalently,

$$y(k) = \sum_{i=-\infty}^{\infty} x(k-i) h_f(i) \quad (A.4)$$

Substituting equation A.4 into A.2 and squaring we have,

$$\begin{aligned} e^2(k) &= d^2(k) - 2d(k) \sum_{i=-\infty}^{\infty} h_f(i)x(k-i) \\ &\quad + [\sum_{i=-\infty}^{\infty} x(i)h_f(k-i)][\sum_{i=-\infty}^{\infty} x(i)h_f(k-i)] \end{aligned} \quad (A.5)$$

Taking the expected value of A.5 we obtain,

$$\begin{aligned} E\{e^2(k)\} &= \psi_{dd}(0) - 2 \sum_{i=-\infty}^{\infty} h_f(i) \psi_{dx}(i) \\ &\quad + \sum_{i=-\infty}^{\infty} h_f(i) \sum_{j=-\infty}^{\infty} h_f(j) \psi_{xx}(j-i) \end{aligned} \quad (\text{A.6})$$

where notation $\psi_{uv}(m)$ denotes the expected value of the product of $u(k)$ and $v(k+m)$. That is,

$$\psi_{uv}(m) = E\{u(k)v(k+m)\} = E\{u(k)v(k-m)\} \quad (\text{A.7})$$

otherwise known as the autocorrelation of $u(k)$ and $v(k)$.

Using variational techniques and letting,

$$h_f(i) = h_{opt}(i) + \epsilon \Delta h(i) \quad (\text{A.8})$$

substituting equation A.8 into equation A.6, taking the partial derivative with respect to epsilon, ϵ , and setting the partial derivative equal to zero we have:

$$\sum_{n=-\infty}^{\infty} \Delta h(n) \left[\sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) - \psi_{xd}(n) \right] = 0 \quad (\text{A.9})$$

The term within the brackets is the discrete form of the Weiner-Hopf equation and must be equal to zero if equation A.9 is to be valid since $\Delta h(n) > 0$ by definition. Therefore,

$$\sum_{i=-\infty}^{\infty} h_{opt}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \quad (\text{A.10})$$

Equation A.10 is the most often encountered in truncated form in linear prediction theory, Pade approximation, adaptive filtering and lattice filtering. The truncated version of A.10 which is generally used in solving the discrete Weiner problem is,

$$\sum_{i=0}^M h_{\text{opt}}(i) \psi_{xx}(n-i) = \psi_{xd}(n) \quad (\text{A.11})$$

or in matrix form,

$$\begin{bmatrix} \psi_{xx}(0) & \psi_{xx}(-1) & \dots & \psi_{xx}(-n) \\ \psi_{xx}(1) & \psi_{xx}(0) & \dots & \psi_{xx}(1-n) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \psi_{xx}(n) & \psi_{xx}(n-1) & \dots & \psi_{xx}(0) \end{bmatrix} \begin{bmatrix} h_{\text{opt}}(0) \\ h_{\text{opt}}(1) \\ \vdots \\ \vdots \\ h_{\text{opt}}(n) \end{bmatrix} = \begin{bmatrix} \psi_{xd}(0) \\ \psi_{xd}(1) \\ \vdots \\ \vdots \\ \psi_{xd}(n) \end{bmatrix} \quad (\text{A.12})$$

APPENDIX B

PROGRAM AND AVERAGE MOVING AVERAGE SIMULATION

THIS PROGRAM GENERATES THE VARIOUS TYPES OF LINEAR SYSTEMS DATA FOR THE ADAPTIVE KALMAN IDENTIFIERS (AKI). THE PROGRAM PERFORMS ALL COMPUTATIONS IN DOUBLE PRECISION AND SIGNS THE OUTPUT DATA ON A CDS FILE. THIS FILE IS ANALYZED BY THE AKI. THE AUTOREGRESSIVE (AR) AND MOVING-AVERAGE (MA) PROCESSES ARE PREDICTED AND DATA IS UNPACKAGED FOR THE AKI.

ROUTINE V(100),V(800),Z(600),V(3000)
RT AL*+ W(800),W(800)

ROUTINE C(100),X(100),A(100),B(100),C(100),D(100),
DOLV = 1.00E-3
SDEV = 1.00E-1

NP = NUMBER OF DATA POINTS FOR INPUT
N = NUMBER OF ADAPTIVE COEFFICIENTS
M = NUMBER OF MA COEFFICIENTS
NL = NUMBER OF OPERATIONS FOR THE AKI
NPER = INITIAL ORDER OF SYSTEM
N=2
M=6
NL=2
NPER=3

WRITE(*,*) N,P,NL

CALL DOLV(DOLV,NP,NL)
CALL DPER(OP(NP,NL,NL))

DO I=1,NP
V(I)=SIG(V(1),W(1))
W(I)=OP(W(1))

A1=V(1)
A2=V(2)
A3=V(3)
A4=V(4)
A5=V(5)
A6=V(6)
A7=V(7)
A8=V(8)

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A₁₂₃₄₅₆₇₈

SOL 101 INITIAT CONDITIONS

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

NR=0

$$\begin{aligned} u_1 &= 0 \\ Y_{(n)} &= A_{1-4} Y_{(K-1)} + A_{2-4} Y_{(K-2)} + A_{3-4} Y_{(K-3)} + A_{4-4} Y_{(K-4)} + A_{5-4} U_{(K-4)} + \\ &A_{6-4} U_{(K-5)} + A_{7-4} U_{(K-6)} + A_{8-4} U_{(K-7)} \\ L_{(n)} &= Y_{(K)} + V_{(K)} \end{aligned}$$

$$KK = KK + 1$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$L_{(n)} = 15.0$$

$$\begin{aligned} L_{(n)} &= 15.0 \\ A_1 &= 2.0 \\ A_2 &= 5.0 \\ u_1 &= 5.0 \\ u_2 &= 10.0 \end{aligned}$$

SOLP
D,L,D

APPENDIX C

POSTCARDS FOR: FIRST TICKET LINE (PLL)

THIS PRACTICALLY ROADS THE MIGRANT FARM MIGRANT FARM PRODUCTS IN THE
BETTER LIFES REALIZED BY THE MIGRANT FARMERS (S&K).
THE SICKENED FARMERS ARE PHYSICALLY AND MENTALLY
WITH THE WITH BROKEN MIGRANT FARMERS SINCE
COSTLY BUT THE MIGRANT IS PAUPERILY SUPPORTED BY
THE LOCALITIES:

$$P_{\alpha\beta}(L^{*\alpha}-1) = \frac{1 + AL^{\alpha}L^{*\alpha}-1 + AL^{\alpha}L^{*\alpha}-2}{1 - AL^{\alpha}L^{*\alpha}-1 - AL^{\alpha}L^{*\alpha}-2} \dots$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200), L(2000), \\ & L(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000), \lambda(2000), \gamma(2000) \\ & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000), L(2000) \end{aligned}$$

INITIAL TICKET LINE

NOTES

$L^{*\alpha}$,
 L^{α}

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200), L(2000), \gamma(2000) \\ & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000) \end{aligned}$$

$$\begin{aligned} & L^{*\alpha}-1 = 2 \\ & L^{*\alpha}-2 \\ & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000) \\ & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200) \end{aligned}$$

$$\begin{aligned} & L^{*\alpha}-1 = 2 \\ & L^{*\alpha}-2 \\ & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200) \end{aligned}$$

$$\begin{aligned} & \text{AL}^{\alpha}L^{*\alpha}-2, L(12), 2000, \lambda(2000), \gamma(2000) \\ & \text{AL}^{\alpha}L^{*\alpha}-1, L(12), 1, \lambda(1200), \gamma(1200) \end{aligned}$$

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0.00001 IN THE PRACTICAL SITUATIONS

CONVERT THE INPUTS TO PLATES AND SET THE PRACTICAL LEVEL
 $\Delta \sigma_{(j)} = (\sigma_{(j)}(0))/1.00001$ WHICH

BLANK INFORMATION LEVEL
IN THE DESIGNATION

PRINTING INPUTS AS A CUMULATIVE INPUT SELECT
 $\Delta \sigma_{(j)} = \text{INPUT}(\Delta \sigma_{(j)})$

FOR THE SIMPLIFIED PLASTICITY MODEL
DATA REQUESTED FOR THE STRENGTH

PRINTING RESULTS OF THE STRENGTH
ANALYSIS ARE PRINTED
FOR THE SIMPLIFIED PLASTICITY MODEL.....INPUT.....

PRINT CUMULATIVE DATA IN FILE d FOR STRENGTH ANALYSIS
 $\Delta \sigma_{(j)} = \text{INPUT}$
 $\Delta \sigma_{(j)} = \text{SOLUT}(1) + \text{STDEV}(1)$
 $\Delta \sigma_{(j)} = \text{SOLUT}(2) + \text{STDEV}(2)$
L65

PRINT CUMULATIVE DATA
 $\Delta \sigma_{(j)} = \text{SOLUT}(3) + \text{STDEV}(3)$
 $\Delta \sigma_{(j)} = \text{SOLUT}(4) + \text{STDEV}(4)$
L66

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Journal of Health Politics, Policy and Law

It is believed that the state park will continue to attract
visitors.

SCHLÜSSELZettel für Pjg

ESTATE PLANNING AND GROWTH OF NET ASSETS

12. *General* (1) $\frac{1}{14}$ (2) $\frac{1}{14}$ (3) $\frac{1}{14}$ (4) $\frac{1}{14}$ (5) $\frac{1}{14}$ (6) $\frac{1}{14}$ (7) $\frac{1}{14}$ (8) $\frac{1}{14}$ (9) $\frac{1}{14}$ (10) $\frac{1}{14}$ (11) $\frac{1}{14}$ (12) $\frac{1}{14}$ (13) $\frac{1}{14}$ (14) $\frac{1}{14}$ (15) $\frac{1}{14}$ (16) $\frac{1}{14}$ (17) $\frac{1}{14}$ (18) $\frac{1}{14}$ (19) $\frac{1}{14}$ (20) $\frac{1}{14}$ (21) $\frac{1}{14}$ (22) $\frac{1}{14}$ (23) $\frac{1}{14}$ (24) $\frac{1}{14}$ (25) $\frac{1}{14}$ (26) $\frac{1}{14}$ (27) $\frac{1}{14}$ (28) $\frac{1}{14}$ (29) $\frac{1}{14}$ (30) $\frac{1}{14}$ (31) $\frac{1}{14}$ (32) $\frac{1}{14}$ (33) $\frac{1}{14}$ (34) $\frac{1}{14}$ (35) $\frac{1}{14}$ (36) $\frac{1}{14}$ (37) $\frac{1}{14}$ (38) $\frac{1}{14}$ (39) $\frac{1}{14}$ (40) $\frac{1}{14}$ (41) $\frac{1}{14}$ (42) $\frac{1}{14}$ (43) $\frac{1}{14}$ (44) $\frac{1}{14}$ (45) $\frac{1}{14}$ (46) $\frac{1}{14}$ (47) $\frac{1}{14}$ (48) $\frac{1}{14}$ (49) $\frac{1}{14}$ (50) $\frac{1}{14}$ (51) $\frac{1}{14}$ (52) $\frac{1}{14}$ (53) $\frac{1}{14}$ (54) $\frac{1}{14}$ (55) $\frac{1}{14}$ (56) $\frac{1}{14}$ (57) $\frac{1}{14}$ (58) $\frac{1}{14}$ (59) $\frac{1}{14}$ (60) $\frac{1}{14}$ (61) $\frac{1}{14}$ (62) $\frac{1}{14}$ (63) $\frac{1}{14}$ (64) $\frac{1}{14}$ (65) $\frac{1}{14}$ (66) $\frac{1}{14}$ (67) $\frac{1}{14}$ (68) $\frac{1}{14}$ (69) $\frac{1}{14}$ (70) $\frac{1}{14}$ (71) $\frac{1}{14}$ (72) $\frac{1}{14}$ (73) $\frac{1}{14}$ (74) $\frac{1}{14}$ (75) $\frac{1}{14}$ (76) $\frac{1}{14}$ (77) $\frac{1}{14}$ (78) $\frac{1}{14}$ (79) $\frac{1}{14}$ (80) $\frac{1}{14}$ (81) $\frac{1}{14}$ (82) $\frac{1}{14}$ (83) $\frac{1}{14}$ (84) $\frac{1}{14}$ (85) $\frac{1}{14}$ (86) $\frac{1}{14}$ (87) $\frac{1}{14}$ (88) $\frac{1}{14}$ (89) $\frac{1}{14}$ (90) $\frac{1}{14}$ (91) $\frac{1}{14}$ (92) $\frac{1}{14}$ (93) $\frac{1}{14}$ (94) $\frac{1}{14}$ (95) $\frac{1}{14}$ (96) $\frac{1}{14}$ (97) $\frac{1}{14}$ (98) $\frac{1}{14}$ (99) $\frac{1}{14}$ (100) $\frac{1}{14}$

SOLVING THE VALUE FUNCTION WITH A

STATEMENT OF THE VICTIMS OF CRIMINAL
LAW INSTITUTIONS
AND THE VICTIMS OF CRIMINAL INSTITUTIONS
OF STATE POLICE AND STATE SECURITY
AGENCIES

12. PRACTICALLY, THE POSITIONING OF THE ALREADY EXISTING STAFFS OF THE
13. POSITIONING OF THE NEW STAFFS OF THE
14. POSITIONING OF THE NEW STAFFS OF THE
15. POSITIONING OF THE NEW STAFFS OF THE

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THE INFLUENCE OF VARIOUS STIMULI ON THE INTESTINAL TRACT 165

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For each $i \in \{1, 2, \dots, n\}$, let $\alpha_i = \frac{1}{n} \sum_{j=1}^n \delta_{x_{ij}}$. Then α_i is a probability distribution on $\{1, 2, \dots, n\}$.

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and the other study sites were located near the water's edge.

APPENDIX D

CARBONATE ROCKS AND SEDIMENTATION IN THE CRETACEOUS OF THE NETHER LAYER (CAK)

PROGRAM ALARM: ADAPTIVE FACTOR: LOAD LIMIT (AKI)

THIS PROGRAM PERIODICALLY CHECKS THE MAIN SYSTEM WHICH IS ADAPTIVE TO THE KALMAN FILTER. THE CAPTURED DATA IS USED TO COMPUTE THE POSITION OF THE CENTER OF GRAVITY OF THE UNKNOWN UNKNOWN SYSTEM. THEREFORE, THIS SYSTEM IS SIMPLY AUGMENTED WITH THE KALMAN FILTER AND THE DOCUMENTATION UPDATES ARE PROVIDED. SUCH DOCUMENTATION IS MAINTAINED IN PAPER FORM. THE SYSTEM WHICH IS TEMPORARILY STORED ON A DISK FILE PROVIDED BY THE ADAPTERS SENSES THE CAPTURED DATA ALSO IN THE MEANTIME. THE OTHER PART OF THE AKI PROGRAM WHICH IS USED FOR MULTIPURPOSE FLIGHT INFORMATION IS THROUGH COMMUNICATON LINE IN THE FORM OF STATEMENTS.

1. www.123rf.com (www.123rf.com) + 123RF Stock Photos

TABLE I.—AVERAGE PERCENTAGE OF INTEGRITY OF SUBCUTANEOUS CARKINOMA TISSUE IN RAGGED STATE AND CALCULATED FROM THE RESULTS OF COOPER ET AL.

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2007-08 NATIONAL CERTIFICATION ACTIVITIES FOR THE
NATIONAL PEDIATRIC CRITICAL CARE WORKER;
SYNTHETIC SYSTEMS. RUM: CERTIFICATE OF COMPLETION

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TABLE II
COMPARISON OF COMPUTED AND INPUT COEFFICIENTS
FOR THE EQUATIONS OF STATE FOR THE VARIOUS
DATA SETS

1911-1915

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$$\text{rank } \left(\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \right) = 0$$

For $\lambda = 0$, we have $\mu_{\lambda} = \mu_0$ and $\nu_{\lambda} = \nu_0$. The terms $(\mu_{\lambda}, \nu_{\lambda})$ in the expansion of $\mu_{\lambda}^{\alpha} \nu_{\lambda}^{\beta}$ are given by

REVIEW OF THE LITERATURE ON UNKNOWN PLANT LIFE

IN THE MOUNTAINS OF ALASKA AND YUKON.

ZARAFI ALLAH

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$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(k) = \int_0^\infty f(x) dx$

CONTENTS | CLOTHES | HAIR | BEAUTY | MAKEUP | LIFESTYLE

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66 11 24

Gardiner et al.

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KUAN (H) V.1(1), JZ(1)

CONVENTIONAL INPUT/OUTPUT DATA INTO BOUNDARY ELEMENT

$$B_{\mu\nu} = \partial_\mu V_{\nu} - \partial_\nu V_{\mu}$$

$$Y(1) = \text{tanh}(\sqrt{2}(1 + \eta(1)))$$

A. J. H. D. L. 1

THE AMERICAN MUSEUM OF NATURAL HISTORY

D. (Int. + Int. C) (Int. + Int.)²⁶

THEORY OF Δ_2 LATTICES 17

the \mathbb{C}^n and \mathbb{C}^{n+1} respectively. The \mathbb{C}^n factor is the complexification of the real manifold M .

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$$\frac{d\psi}{d\lambda} = \frac{1}{\lambda} \frac{d\psi}{d\mu} + \frac{1}{\lambda^2} \frac{d^2\psi}{d\mu^2}$$

THE INFLUENCE OF THE ENVIRONMENT ON THE GROWTH OF COTTON

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CHINESE LITERATURE IN THE 1930S 11

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$$v_0(\alpha) = \left(v_0(\alpha_{\text{sat}}) \right) \delta v(1)$$

C. K. Hsu

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LOCATE INITIAL VALUES OF THE PASTORAL VECTORS

IN $(K+1) \times 1$ TO 40

MOVE FIRST PASTORAL VECTOR WITH $Y(K+1)$ ELEMENTS

TO $\hat{L} = L^{(K+1)}$

$= C \cdot R(1,1) + Y(K+1) \cdot f_{K+1}$

40 construct
 $L^k = R(k,1) + Y(k) \cdot f_k$

Locate first non-zero vector with $Y(K+1)$ ELEMENTS

IN $(K+1) \times 1$

$\hat{L} = L^{(K+1)} = Y(K+1) \cdot C(1-M)$

IF $C(1-M) > 0.001$ GO TO 60

IF $C(1-M) \leq 0.001$ MOVE FIRST NON-ZERO ELEMENT, $Y(K+1)$ TO
 $R(1,1) = X(K+1)$

$R(1,1) = 1.00$

$Y(K+1) = GO(1+M, Y(K+1))$

$R(1,1) = 0.001$

Locate first non-zero vector with $Y(K+1)$ ELEMENTS
 $L^k = R(k,1) + Y(k)$

$R(1,1) = 1.00$

$L^k = R(k,1) + Y(k) \cdot C(1-M)$

IF $C(1-M) > 0.001$ GO TO 60

IF $C(1-M) \leq 0.001$ MOVE FIRST NON-ZERO ELEMENT, $Y(K+1)$ TO $R(1,1)$

$R(1,1) = 1.00$

$Y(K+1) = GO(1+M, Y(K+1))$

$R(1,1) = 0.001$

Locate first non-zero vector with $Y(K+1)$ ELEMENTS
 $L^k = R(k,1) + Y(k)$

$R(1,1) = 1.00$

$Y(K+1) = GO(1+M, Y(K+1))$

$R(1,1) = 0.001$

$L^k = R(k,1) + Y(k)$

$R(1,1) = 1.00$

$Y(K+1) = GO(1+M, Y(K+1))$

$R(1,1) = 0.001$

CONTRACT NUMBER = YGCH+NGH1-N-1

THE CONTRACTOR WILL VICTOR WES NEW BRUN SWAC IN

44 POINT (6,42)
44 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0
42 POINT (6,42)

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0
42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

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42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

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42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

42 POINT (6,42) (0,11), H.V.C. (0,11) IS: 0/0

$\sum_{j=1}^J \frac{1}{\rho_j} \frac{\partial \ln N_{ij}}{\partial \rho_j} \left(\frac{\partial \ln N_{ij}}{\partial \rho_j} \right)_{\rho_j=1}, \forall i, j \in \{1, \dots, M\}$

occurs from the battle line

$$\begin{aligned} \text{loss}_{ij} &= k_{ij} \\ \text{loss}_{ij} &\leq \text{loss}_{i'j'} \quad \forall i' \neq i, j' \neq j \\ \text{loss}_{ij} &\geq \text{loss}_{i'j'} \end{aligned}$$

contribute to sales, costs, and future earnings, given (k) & present (k') cash flows

contribution margin for the new clients
cash return

relative market value coefficients are new computing
the sales, cost and ready for printing

new clients, $i = 1, \dots, M$ to 10)

absolute market values and losses
 $k_{ij} = k_{i'j'}$

$\text{loss}_{ij} = k_{ij} - k_{i'j'}$, and
 $\text{loss}_{ij} = \text{loss}_{i'j'}$ if $k_{ij} = k_{i'j'}$

total market value $= \sum_{i,j} (\text{loss}_{ij} (1, k_{ij}))$

total market value (1, k_{ij})

total market value $= \sum_{i,j} (\text{loss}_{ij} (1, k_{ij})) + \sum_{i,j} (\text{loss}_{ij} (1, k_{ij}))$
 $\text{loss}_{ij} (1, k_{ij}) = \text{loss}_{ij} (1, k_{ij})$
 $\text{loss}_{ij} (1, k_{ij}) = \text{loss}_{ij} (1, k_{ij})$
 $\text{loss}_{ij} (1, k_{ij}) = \text{loss}_{ij} (1, k_{ij})$

total market value $= \sum_{i,j} (\text{loss}_{ij} (1, k_{ij}))$

$\text{loss}_{ij} (1, k_{ij})$

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Figure 5 shows the results of the PES calculations for the $\text{C}_2\text{H}_2 + \text{NO}$ reaction at different temperatures. The calculated rate coefficients are plotted as a function of temperature. The calculated rate coefficient is in good agreement with the experimental data.

RECEIVED DEPT OF WORKING MACHINES NO 1724 LETTER PRELIMINARY

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It is to be observed that the loci

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14. $\text{H}_2\text{O} + \text{C}_2\text{H}_5\text{OH} \rightarrow \text{CH}_3\text{COOH} + \text{H}_2\text{O}$
 15. $\text{H}_2\text{O} + \text{C}_2\text{H}_5\text{OH} \rightarrow \text{CH}_3\text{COOH} + \text{H}_2\text{O}$

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For example, $\text{P}(\text{A} \cap \text{B}) = \text{P}(\text{A}) \cdot \text{P}(\text{B} | \text{A})$

$$P_{k+1}(x/k+1) = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)} = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)}$$

1.0. $P_{k+1}(x/k+1) = \frac{1}{G(1, k+1)} \cdot \frac{1}{G(1, k+2)} \cdot \frac{1}{G(1, k+3)} \cdot \dots \cdot \frac{1}{G(1, k+1+k-1)}$

1.1. $x/k+1 = \frac{x}{k+1} + \frac{k}{k+1}$ Then we have $x/k+1 = x/k + 1$

1.1.4. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.5. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.6. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.7. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.8. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.9. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.10. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.11. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.12. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.13. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.14. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.15. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.16. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.17. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.18. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.19. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.20. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.21. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.22. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.23. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.24. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.25. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.26. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.27. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.28. $P_{k+1}(x/k+1) = P_k(x/k)$

1.1.29. $P_{k+1}(x/k+1) = P_k(x/k)$

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ALG 54145

ALG 54146

ALG 54147

ALG 54148

Color CRT: $\text{shk}(\text{kr}, \text{ks}+1) = \text{shk}(\text{kr}) * \text{shk}(\text{ks})$

32. *GRADIENT*

33. $\frac{\partial}{\partial x} \text{values at } (k+1) = \text{Ainst}(1,k) + \epsilon(1,F+1) * t.b$
34. $\frac{\partial}{\partial x}$
35. $\frac{\partial}{\partial y}$

36. $\text{SOLUTIVE EQUATION } (\text{A}, N, P)$
37. $\text{THIS SOURCE IS FOR THE PRESENT USE OF THE STIRLING CIRCLE MAP}$
38. $\text{WITH THE FURTHER REQUIREMENT THAT THE KNOTS }$

39. $\text{ARE KNOWN AT }(1,1), (2,1), \dots, (N, 1)$

40. $\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \frac{\partial}{\partial w}$
41. $\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \frac{\partial}{\partial w}$
42. $\frac{\partial}{\partial x} (1,1) = \frac{\partial}{\partial y} (1,1)$

43. *STURGEON*
44. *TRAP*

45. $\text{SOLUTIVE EQUATION COMPUTED BY THIS PROGRAM IS FOR THE PRESENT USE OF THE STIRLING CIRCLE MAP}$
46. $\text{WITH THE FURTHER REQUIREMENT THAT THE KNOTS }$
47. $\text{ARE KNOWN AT }(1,1), (2,1), \dots, (N, 1)$
48. AND THE KNOTS

49. $\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \frac{\partial}{\partial w}$
50. $\frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \frac{\partial}{\partial w}$
51. $\frac{\partial}{\partial x} (1,1) = \frac{\partial}{\partial y} (1,1)$
52. $\frac{\partial}{\partial x} (1,1) = \frac{\partial}{\partial z} (1,1)$
53. $\frac{\partial}{\partial x} (1,1) = \frac{\partial}{\partial w} (1,1)$
54. $\frac{\partial}{\partial y} (1,1) = \frac{\partial}{\partial z} (1,1)$
55. $\frac{\partial}{\partial y} (1,1) = \frac{\partial}{\partial w} (1,1)$
56. $\frac{\partial}{\partial z} (1,1) = \frac{\partial}{\partial w} (1,1)$

obtained by the following steps:

1. Let \mathcal{N} be the set of nodes in G and M be the set of edges in G .

$$\text{Let } \mathcal{E} = \{(e_1, e_2) : e_1, e_2 \in M\}$$

$$\text{Let } \mathcal{E}' = \{(e_1, e_2) : e_1, e_2 \in M, e_1 \neq e_2\}$$

2. Let

$$\text{Let } \mathcal{E}'' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}, e_1 \cap e_2 \neq \emptyset\}$$

3. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

4. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

5. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

6. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

7. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

8. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

9. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

10. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

11. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

12. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

13. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

14. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

15. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

16. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

17. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

18. Let $\mathcal{E}''' = \{(e_1, e_2) : e_1, e_2 \in \mathcal{E}'', e_1 \cap e_2 \neq \emptyset\}$

This function takes a vector \mathbf{v} and a matrix M as input and returns the result of the multiplication $M \cdot \mathbf{v}$. It uses a loop to iterate through each row of M and multiply it by the corresponding element of \mathbf{v} . The result is stored in a temporary variable $temp$ and then added to the current value of $result$. Finally, the function returns the total result.

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SOON TO FOLLOW. (144)
AND, CONSIDERED, AND, CONSIDERED, IN, THE, MEANTIME, AT, THE, PRESENT,
LEVEL, THE, (K-1) . . . Y(N-N), U(K) . . . U(N-N)

LEVEL, SO, AS, K, GROWS, AND, SO, DOES, U(K), AND, SO, DOES, U(N-N),
WHICH, IS, AS, K, GROWS, AND, SO, DOES, U(N-N), PERIODICALLY, (20, 23),
WHICH, IS, AS, K, GROWS, AND, SO, DOES, U(N-N), PERIODICALLY, (20, 23),
WHICH, IS, AS, K, GROWS, AND, SO, DOES, U(N-N), PERIODICALLY,

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

SOON, LEVEL, THE, (K-1) . . .

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

SOON, LEVEL, THE, (K-1) . . .

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

SOON, LEVEL, THE, (K-1) . . .

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

SOON, LEVEL, THE, (K-1) . . .

SOON, LEVEL, THE, (K-1) . . .
IN, THE, (K-1) . . . U(K-1) . . .

THE AFRICAN-AMERICAN POPULATION IN THE UNITED STATES

$$L_{\mu\nu} = \frac{1}{2} \partial_\mu \partial_\nu L - \frac{1}{2} g_{\mu\nu} \nabla^\lambda \nabla_\lambda L + \frac{1}{2} g_{\mu\nu} R - g_{\mu\nu} \left(\frac{1}{2} R^2 + \frac{1}{2} \Lambda \right)$$

See *W. H. Dall, 1882*, pp. 14-47.

卷之三

$$\frac{d\psi}{dt} = \frac{1}{2} \left(\frac{\partial \psi}{\partial x} \right)^2 + \frac{1}{2} \left(\frac{\partial \psi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x^2} \right) + \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial t^2} \right) - \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x \partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial^2 \psi}{\partial x \partial t} \right)^2$$

11. *Antarcticus* (L.) *Scopoli*

$$\begin{aligned} & \frac{1}{1-x} + \frac{1}{1+x} = \frac{2}{1-x^2}, \\ & \frac{1}{1-x} - \frac{1}{1+x} = \frac{2x}{1-x^2}, \\ & \frac{1}{1-x^2} = \frac{1}{1-x} + \frac{1}{1+x}. \end{aligned}$$

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MENTAL STATE (80) POINTS OUT IT WOULD TAKE ANOTHER POINT TO GET THE POINTS

1. *Environ. Monit. Assess.* 1998, 47, 11-18. © 1998 Kluwer Academic Publishers. Printed in the Netherlands.

α	β	γ	δ	ϵ
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1

卷之三

故人不以爲子也。子之不孝，則無子矣。故曰：「子不孝，無子也。」

卷之三

卷之三

APPENDIX E

THIS PROGRAM COMPUTES THE MOVING-AVERAGE(MA) FOR EQUIVALENTLY WEIGHTED TRANSVERSAL FILTER COEFFICIENTS. IF A LEAST MEANS SMOOTHING FILTER USING AN ADAPTIVE ALGORITHM WHICH IMPLEMENTS THE ACTUAL STEEPEST DESCENT. THIS ALGORITHM WAS FIRST DEVISED BY KIRKCH.

KELAL #6 V(2000), K(12), X(12), U(2000), F, KS, D, C
KELAL #4 V(12000), Y(12000), Z(12000), W(12,2000)

THE CIVIL SERVICE FACULTY IS KS
KS = -Z₁CD-1

WRITE (6,10) KS
100 PRINT '(1X,1X,28) THE CONVERGENCE FACTOR: KS = ,F10.4,/)
101 USES OF THE FILTER IS M
102 READ (5,103) K

20.0 PERCENTILE ($t_{1200}^{(20)}(15,2x)$, ZINWEICHTS AKE ESTIMATED, /)

INT DIAGNOSTIC FLAG IS FLAG
LINK = 0

LIT. KUMON ET ITERATIONS IS NP
 $N^P = 100$

卷之三

$$1 \in \lambda \wedge (1) = 0.050$$

$$\text{det} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 1 \cdot 1 \cdot 1 = 1$$

DO N=1,40 K=2, Np

$$U = Y(K-1)$$

DO I=1,10
W(I)=((L1*A(I)+B(I))/C(I))
DO J=1,5
X(J)=D(J)*SIN(C(I))
CONTINUE

IF (L1.EQ.0.0) GO TO 80

W(1)=((L1*A(1)+B(1))/C(1))*X(1); X(1)=0.0

DO I=2,N
W(I)=((L1*A(I)+B(I))/C(I))*X(I); X(I)=0.0

DO J=1,5
X(J)=D(J)*SIN(C(I)); X(I)=0.0

CONTINUE

CALL RMLLI(W,X,A,C)
END-C

W(1)=((L1*A(1)+B(1))/C(1))*X(1); X(1)=0.0
DO I=2,N
W(I)=((L1*A(I)+B(I))/C(I))*X(I); X(I)=0.0

DO J=1,5
X(J)=D(J)*SIN(C(I)); X(I)=0.0

CONTINUE

IF (L1.EQ.-1)
DO I=1,M
W(I)=SIN(L1*A(I))

IF (L1.EQ.0.0) GO TO 50

W(1)=((L1*A(1)+B(1))/C(1))*X(1); X(1)=0.0

DO I=2,N
W(I)=((L1*A(I)+B(I))/C(I))*X(I); X(I)=0.0

DO J=1,5
X(J)=D(J)*SIN(C(I)); X(I)=0.0

CONTINUE

COMPUTE W(K+1): THE NEXT STEP PREDICTION

$$\begin{aligned} W(K+1) &= L1 * A(K+1) \\ W(K+1) &= W(K) - X(X(K)) \end{aligned}$$

SUB 1 THE INPUT DATA VECTOR THRU THE DELAY LINE BY ONE
L = K+1
N = K-1
DO I=1,M
L(I) = L-1
TO A(L,I) = U(K)
A(L,I) = U(K-1)

LSC0520
LSC0530
LSC0540
LSC0550
LSC0560
LSC0570
LSC0580
LSC0590
LSC0600
LSC0610
LSC0620
LSC0630
LSC0640
LSC0650
LSC0660
LSC0670
LSC0680
LSC0690
LSC0700
LSC0710
LSC0720
LSC0730
LSC0740
LSC0750
LSC0760
LSC0770
LSC0780
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LSC0920
LSC0930
LSC0940
LSC0950
LSC0960
LSC0970
LSC0980

4.0 CERN Input

6.0 $F = NP - 1$
6.0 R11 E(4;160) R1,NP,P
16C PERAT (2;15)

6.0 I 170 I = 1, P
6.0 I 70 J = 1, NPP
170 PERAT (4;130) NW(1,J)
18C PERAT (4;20,5)

SUP
HNU

LPS C1010
LPS C1020
LPS C1030
LPS C1040
LPS C1050
LPS C1060
LPS C1070
LPS C1080
LPS C1090
LPS C1100
LPS C1110
LPS C1120
LPS C1130
LPS C1140

SUBROUTINE MULT(B,C,M,D)
REAL A,B,C,D
DIMENSION B(12),C(12)

INTEGER M

D=0.0
DO 10 I=1,M
D=D+B(I)*C(I)
10 RETURN

APPENDIX F

151

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```

203 PRINTAI (/,13X,'ONLY THE LAST',13,' ITERATIONS ARE PRINTED',/)

204 READ FOPEN,111,111,1,10X,5HERRR,5X,6HOUTPUT,1)
111 10L KNUMBER OF ITERATIONS IS NP
NP = 4000
NP+1 = NP+1
NP-1 = NP-1
111 11L 4 (NP+1,I,2),UR,(N*LT,1) GO TO 205

205 D0 1,0 I=1,NM
  A(I) = 0.000
  X(I) = 0.000
  111 XX(I) = 0.000
  111 12L 1,1 I=1,N
  D(I) = 0.000
  Y(I) = 0.000
  111 13L 2,0 I=1,NP
  D0 KFAL (I,J) V1(I),V2(I)
  13L 14L 3,0 I=1,NP
  D(I) = D(I-1)
  U(I) = LDE(V1(I+1))
  SC Y(I) = LDE(V2(I+1))
  14L 15L 4,0 K=2,NP
  D = Y(K-1)
  K3 = K-1

152 16L 1,1 DIAU;E0;0) GO TO 30
  WRITE (6,122)'COMPUTE THE ERRORS WITH',/)

162 17L FOPEN (6,123)'COMPUTE THE ERRORS WITH',/)

172 18L READ (6,124)'ERR',10X,A(I):',10X,Y(I):',/)

182 19L READ (6,125)'A(I)',A(I)
  192 20L READ (6,126)'X(I)',X(I)
  202 21L READ (6,127)'B(I)',B(I)

```



```

C          WRITE (6,64) '(A(I),I=1,M)
C          WRITE (6,64) '(A(I),I=1,M)
C          WRITE (6,65) 'NITER UPDATE: 0,/,10X,0B(1):0,/')
C          WRITE (6,64) '(B(I),I=1,N)
C          CURINL
C
C          SUBJ THE INPUT DATA VECTOR THRU THE DELAY LINE BY ONE
C          IN (P,PC,1) CC T,J 72
C
C          L1 = N+1
C          N2 = N-1
C          DO I=L1-1,N1
C          L1 = L1-1
C          X(L1) = X(L1-1)
C
C          L2 = L1+1
C          L3 = N-1
C          DO J=L1,L3
C          L4 = L2-1
C          Y1(I,J) = Y1(L4-1)
C
C          CONFLAT
C
C          WRITE (6,100) M,NPPL
C          WRITE (6,100) N2,NPPL
C          WRITE (6,100) P1
C
C          DO 170 I=1,M
C          DO 170 J=1,NPPL
C          170 WRITE (4,185) AA(I,J)
C
C          DO 171 I=1,M
C          DO 171 J=1,NPPL
C          171 WRITE (4,185) BB(I,J)
C
C          WRITE (6,201) ((3LA(I,J),I=1,M)
C          WRITE (6,201) ((3LA(I,J),I=1,M)
C
C          DO 202 I=1,M
C          DO 202 J=1,M
C          202 WRITE (4,203) ((AA(I,J),I=1,M)
C
C          DO 203 I=1,M
C          DO 203 J=1,M
C          203 WRITE (4,203) ((BB(I,J),I=1,M)

```

C WRITE (6,201) (DB(I), I=1,N5)
C 201 L=NPR1, NPP
C 202 WRITE (6,203) DB(I,L), I=1,N5
C 203 M TO 206
C 205 WRITE (6,207)
C 207 NUMERATOR IS INCORRECT; EITHER THE NUMERATOR OR THE DENOMINATOR IS INCORRECT; EITHER M OR N IS INCORRECTLY SPECIFIED. READ IN THE CORRECT ./5X VALUES FOR M AND N AND TRY AGAIN.
C 206 CONTINUE
C STOP
C END

LPS C1760
LPS C1970
LPS C1980
LPS C1990
LPS 02000
LPS 02010
LPS 02020
LPS C2020
LPS C2030
LPS C2040
LPS C2050
LPS C2060
LPS C2070
LPS C2080
LPS 02090
LPS C2100
LPS C21100
LPS C2120

SUBROUTINE WHDL (A,B,N,D)
RET * J E (12) * E (12), J
INT GR N
L=0,000
L=1,1=N
D=L,A(1)*D(1)
RT JRN
END

SC2150
SC2160
SC2170
SC2180
SC2190
SC2200
SC2210
SC2220
SC2230
SC2240
SC2250
SC2260
SC2270

SOURCELIST FILE (A,B,N,E)
READ *B, A(L2), B(L2), D
IF,IT,GTR, N
C=C, CLO
DO,10,I=2,N
10 D=L+A(I)+B(I)
RETURN
END

LPS02300
LPS02310
LPS02320
LPS02330
LPS02340
LPS02350
LPS02360
LPS02370
LPS02380
LPS02390
LPS02400

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